SECTION 1

Modeling in Contemporary Software Programs

1.1 General

The numerical tools used by expert engineers to analyze seismically isolated structures at the time of this writing represent the state of practice. This section describes how elastomeric and sliding isolators are modeled in computer codes that are widely used in the United States, noting that numerical models of seismic isolation systems should a) include all isolators in the seismic isolation system, and b) the account for the spatial distribution of the isolators across the plan footprint of the isolated structure.

There are two ways to represent the physical model of an isolator: 1) a full three-dimensional continuum model, and 2) a three-dimensional discrete model in which two nodes are connected by six springs to represent the mechanical behavior in each of the six directions (three translation and three rotation).

General-purpose Finite Element Analysis (FEA) programs such as ABAQUS (Systèmes, 2010a), LS-DYNA (LSTC, 2012a) and ANSYS (System, 2011), enable the use of discrete and continuum models of seismic isolators. The special purpose software programs used for structural analysis of base-isolated structures such as SAP2000 (Wilson, 1997), OpenSees (McKenna *et al.*, 2006), PERFORM-3D (CSI, 2006), and 3D-BASIS (Nagarajaiah *et al.*, 1989), model an elastomeric bearing as a two-node discrete element with stiffness in each of the six principal directions represented by linear or nonlinear springs between the two nodes. Analytical expressions for force and stiffness can be used to define a spring in any direction.

The modeling techniques for different types of isolator in seven software programs (SAP2000, OpenSees, PERFORM-3D, 3D-BASIS, LS-DYNA, ABAQUS, and ANSYS) are discussed in the following sections. Six types of isolator are considered: 1) Low Damping Rubber (LDR) bearing, 2) Lead Rubber (LR) bearing, 3) single Friction Pendulum (FP) bearings, 4) double FP bearings, 5) triple FP bearings, and 6) XY-FP bearings. The modeling techniques discussed for these isolators can be extended to other types of isolator.

1.2 SAP2000

In SAP2000, isolators are modeled using Link/Support element option. Link is a two-node element connected by six springs. Each node has six degrees of freedom. A description of the Link element is given in Figure 1.1.

SAP2000 provides the option to use the link element to model any structural element that can be represented as 2-node element. The property data form for the link element is shown in Figure 1.2. Low damping rubber (LDR), lead rubber (LR), flat sliders, single Friction Pendulum (FP), and double FP bearings can be modeled using the link element. The shearing behavior is based on the model proposed by Park *et al.* (1986) and extended for seismic isolation bearings by Nagarajaiah *et al.* (1991). For nonlinear force-deformation response, either a) elastic and post-elastic stiffness values, or b) equivalent linear stiffness, is assigned.

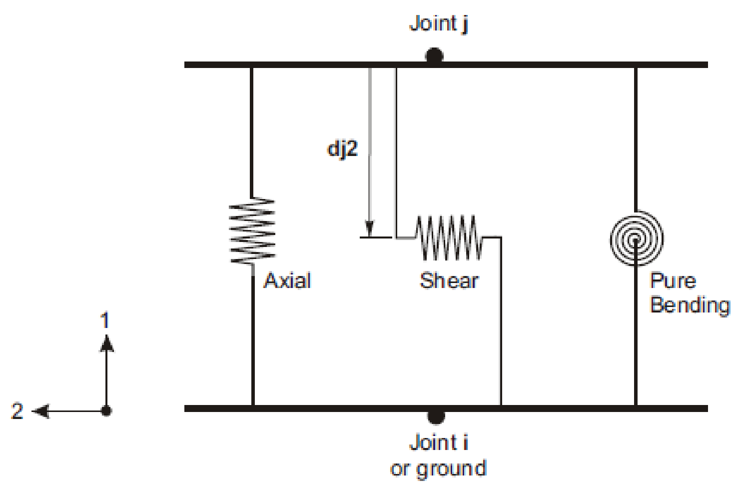
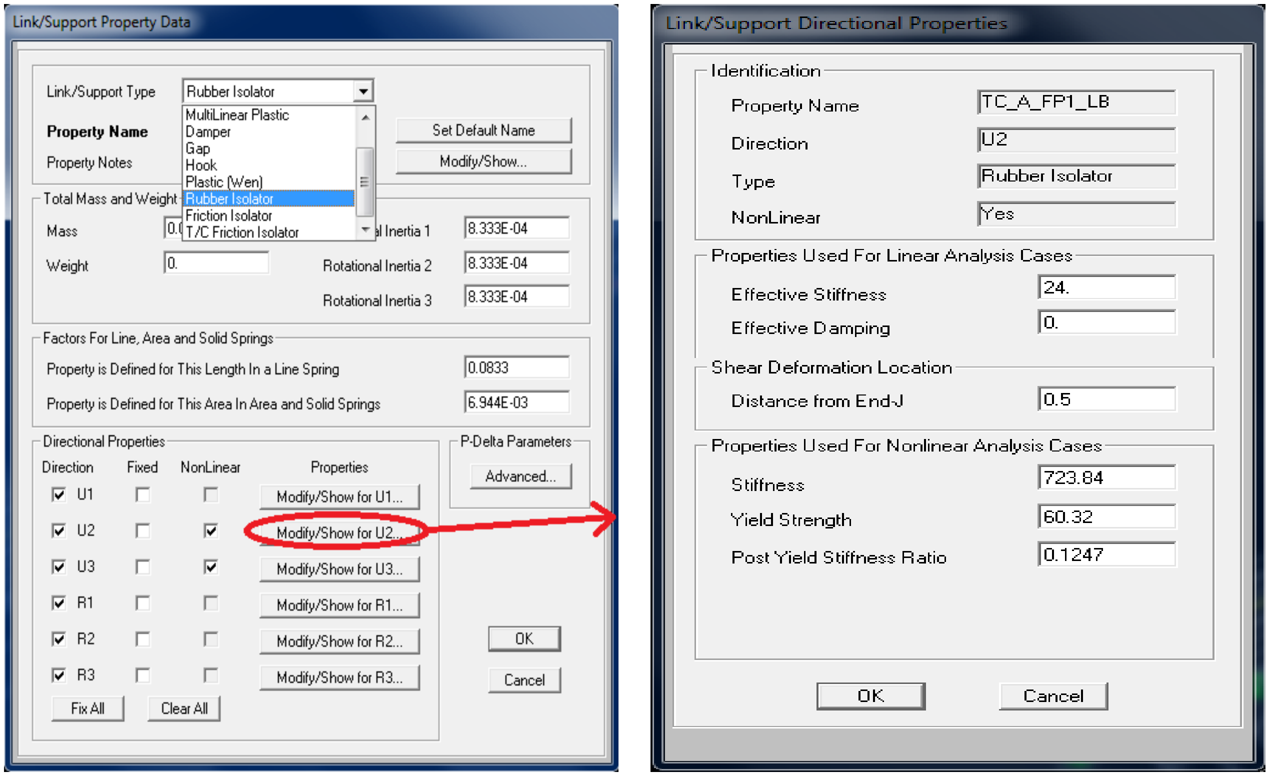


Figure 1.1: Three of the six independent springs in a Link/Support element



a) Link/support property data form **b) Nonlinear directional properties for U2**

Figure 1.2: Link/Support property data input to SAP2000 (CSI, 2011)

For the *elastomeric bearing* (rubber isolator) option in the link element, nonlinear (bilinear) properties can be assigned to the two horizontal shear directions, but only linear elastic behavior is accommodated for the remaining axial and three rotational directions.

The *friction isolator* option allows compression-only behavior in the axial direction, similar to a gap element, and has coupled frictional properties in the two horizontal shear directions. The “T/C friction isolator” option is intended for XY-FP bearing with uplift restraint (Marin-Artieda *et al.*, 2009; Roussis and Constantinou, 2005). This option allows modeling of both tension and compression behavior in the axial direction, and behavior in two shear directions are uncoupled. The axial behavior in both types of friction isolator is always nonlinear. Each shear deformation degree of freedom can be independently assigned to either linear or nonlinear behavior. Both elements for friction-based isolators consider the velocity dependence of the friction co-efficient, but the pressure and temperature dependency of the coefficient of sliding friction is not considered.

SAP2000 does not provide an element of the triple Friction Pendulum bearings. Series and parallel models have been developed using combination of Link and beam elements to simulate the behavior of these bearings in SAP2000. Fenz and Constantinou (2008) and Sarlis and Constantinou (2010) provide detailed information on series and parallel models, respectively.

1.3 3D-BASIS

The computer program 3D-BASIS (also 3D-BASIS-M, 3D-BASIS-TABS, and 3D-BASIS-ME-MB) is used for the nonlinear dynamic analysis of seismically isolated structures (Nagarajaiah *et al.*, 1989; Tsopelas *et al.*, 2005). The analysis model and reference frames of a base-isolated structure in 3D-BASIS-ME-MB are shown in Figure 1.3 and Figure 1.4. The software program provides the option to use elastomeric (LDR and LR) bearings and friction-based isolators that include the single FP bearing, the double FP bearing, and the XY-FP bearing. Sarlis *et al.* (2009) added an element for modeling triple FP bearing in 3D-BASIS.

The isolators in 3D-BASIS are modeled using explicit nonlinear force-displacement relationships. The isolators are considered rigid in vertical direction and do not offer any torsional resistance. The following elements are available in the program 3D-BASIS-ME-MB for modeling the behavior of isolators (Tsopelas *et al.*, 2005):

1. Linear elastic element.
2. Linear and nonlinear viscous elements for fluid viscous dampers or other devices displaying viscous behavior.
3. Hysteretic element for elastomeric bearings and steel dampers.
4. Stiffening (biaxial) hysteretic element for elastomeric bearings.
5. Hysteretic element for flat sliding bearings.
6. Hysteretic element for friction pendulum bearings.
7. Hysteretic element for the uplift-restraining FP (XY-FP) bearings.

The model proposed by Park *et al.* (1986) and extended for analysis of seismic isolators by Nagarajaiah *et al.* (1989) is used for the unidirectional and bi-directional hysteretic elements. The velocity dependence of friction is considered. Uplift is considered indirectly by assigning zero axial force to the isolator in which uplift occurs and redistributing axial forces to the other isolators to maintain vertical equilibrium.

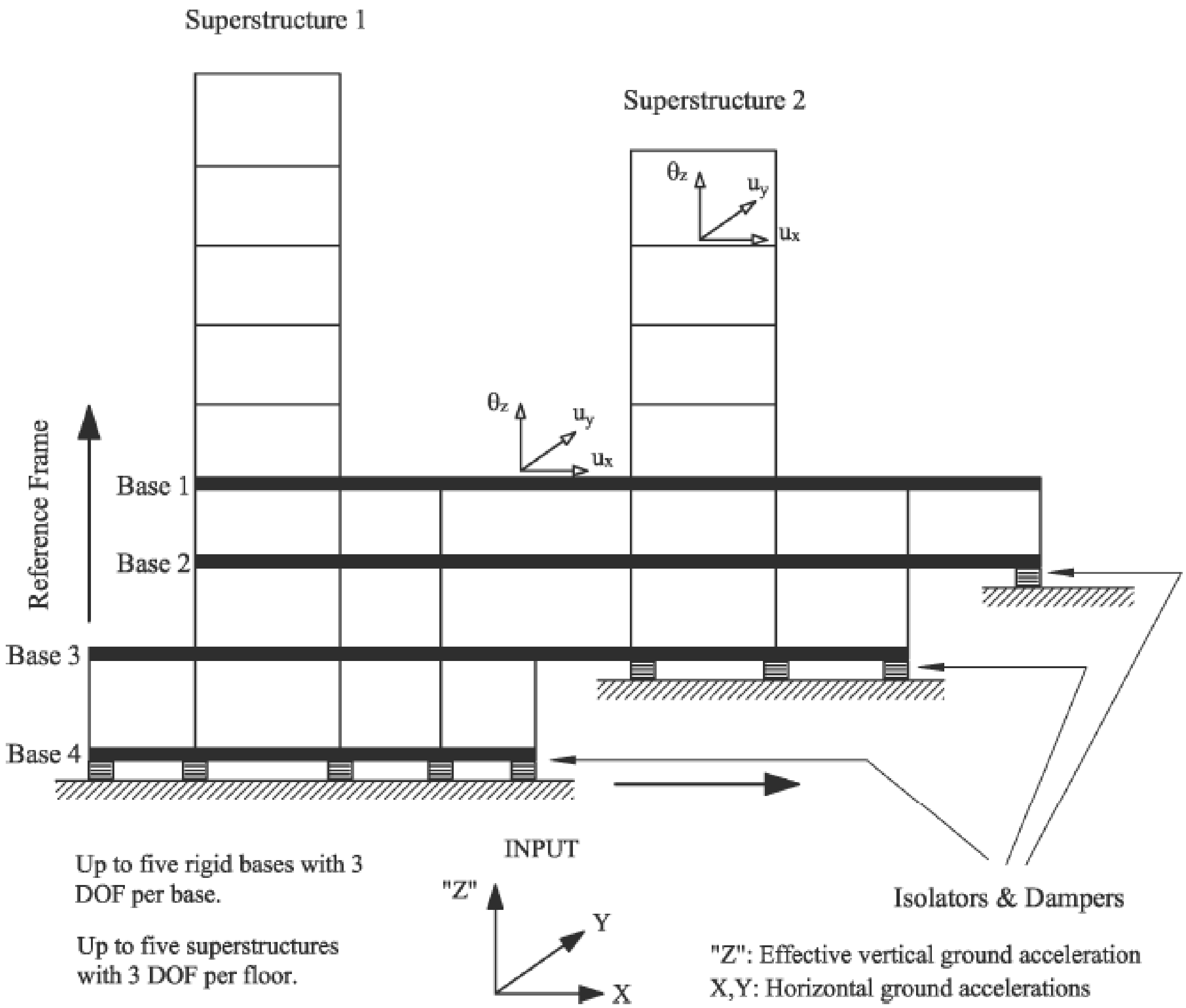


Figure 1.3: Model that can be analyzed in 3D-BASIS-ME-MB (Tsopelas et al., 2005)

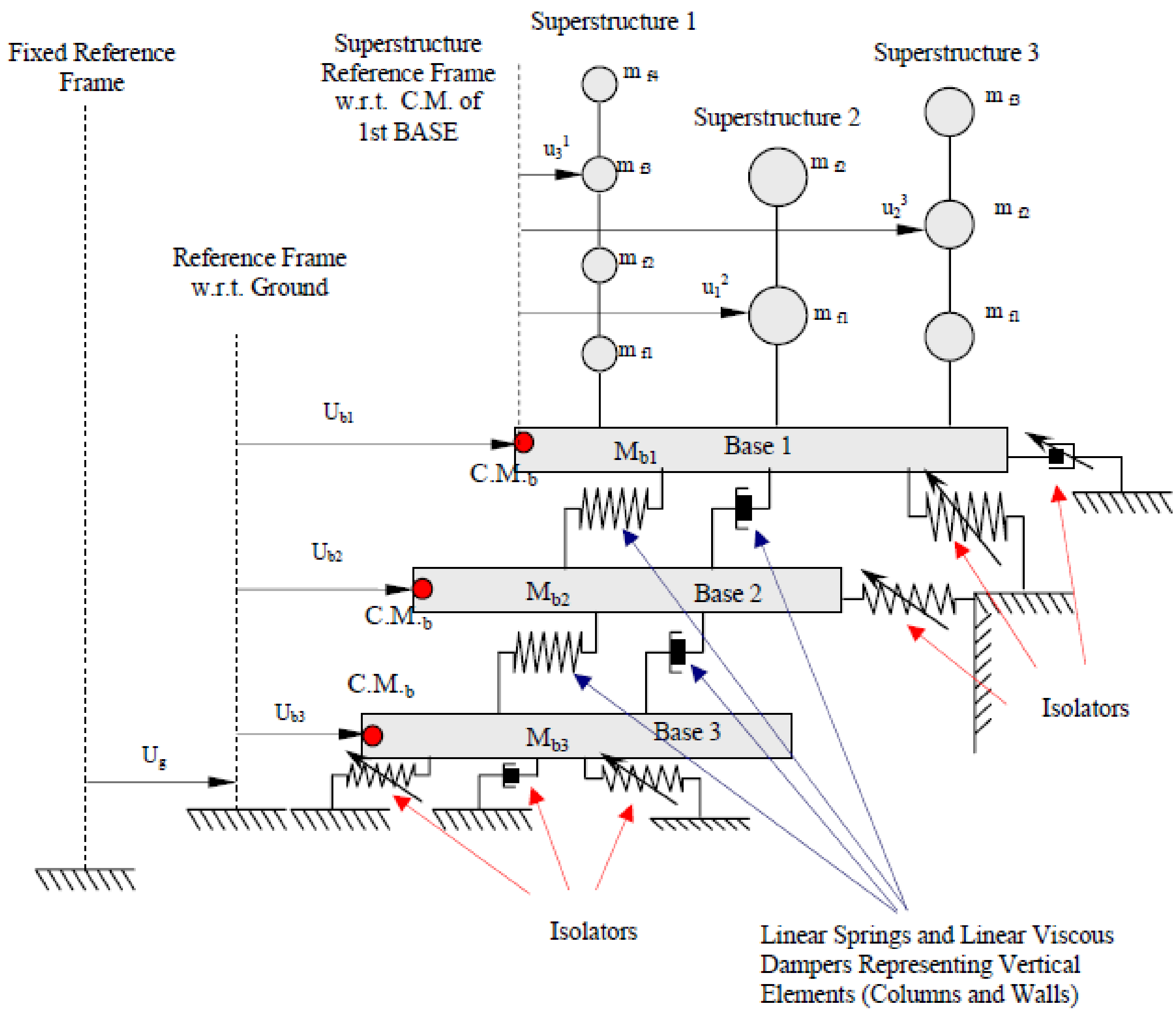


Figure 1.4: Degrees of freedom and reference frames in 3D-BASIS-ME-MB (Tsopelas et al., 2005)

1.4 PERFORM-3D

PERFORM-3D is a software program used for the nonlinear dynamic analysis of structures. PERFORM-3D has powerful capabilities for inelastic analysis. The nonlinear model proposed by Park *et al.* (1986) and extended for analysis of seismic isolators is used in two horizontal (shear) directions. Elastic stiffness is used in the vertical (axial) direction with option to provide different values in compression and tension. The local-axis orientation of the isolators must be assigned to seismic isolator elements in PERFORM 3D. Axis 3 is usually defined as the vertical (axial) direction of an isolator and Axes 1 and 2 are the horizontal (shear) directions.

1.5 ABAQUS, LS-DYNA, and ANSYS

1.5.1 Continuum Modeling

In the continuum modeling approach, an isolator is modeled as a three-dimensional continuous object with appropriate material and geometrical properties assigned to different components of the isolator. All three FEA software programs, ABAQUS, LS-DYNA, and ANSYS use a similar approach to model isolators. The wide range of capabilities of FEA allows a user to model complex phenomena like heating of the lead core in LR bearing and frictional properties of FP bearings. The capability of a FEA model of an isolator to simulate the actual behavior depends on how detailed a model is constructed. Very detailed models will increase the computational effort and isolation systems incorporating them may not be amenable to the analysis of large base isolated structures in which many isolators must be modeled explicitly.

An elastomeric bearing is modeled as a multilayer object of alternating rubber and steel. Rubber layers are usually meshed using solid elements and the steel layers can be modeled using either solid or shell elements. A finite element model of an elastomeric bearing constructed in ABAQUS is shown in Figure 1.5.

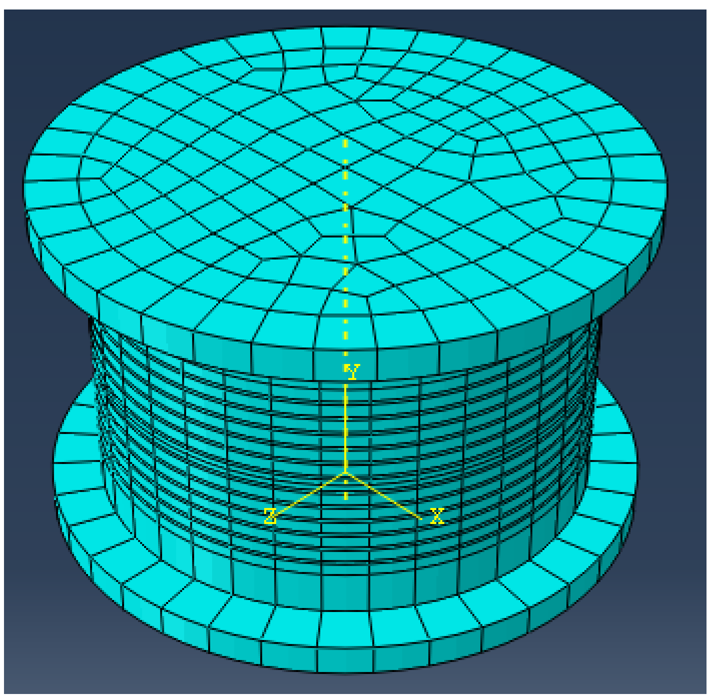
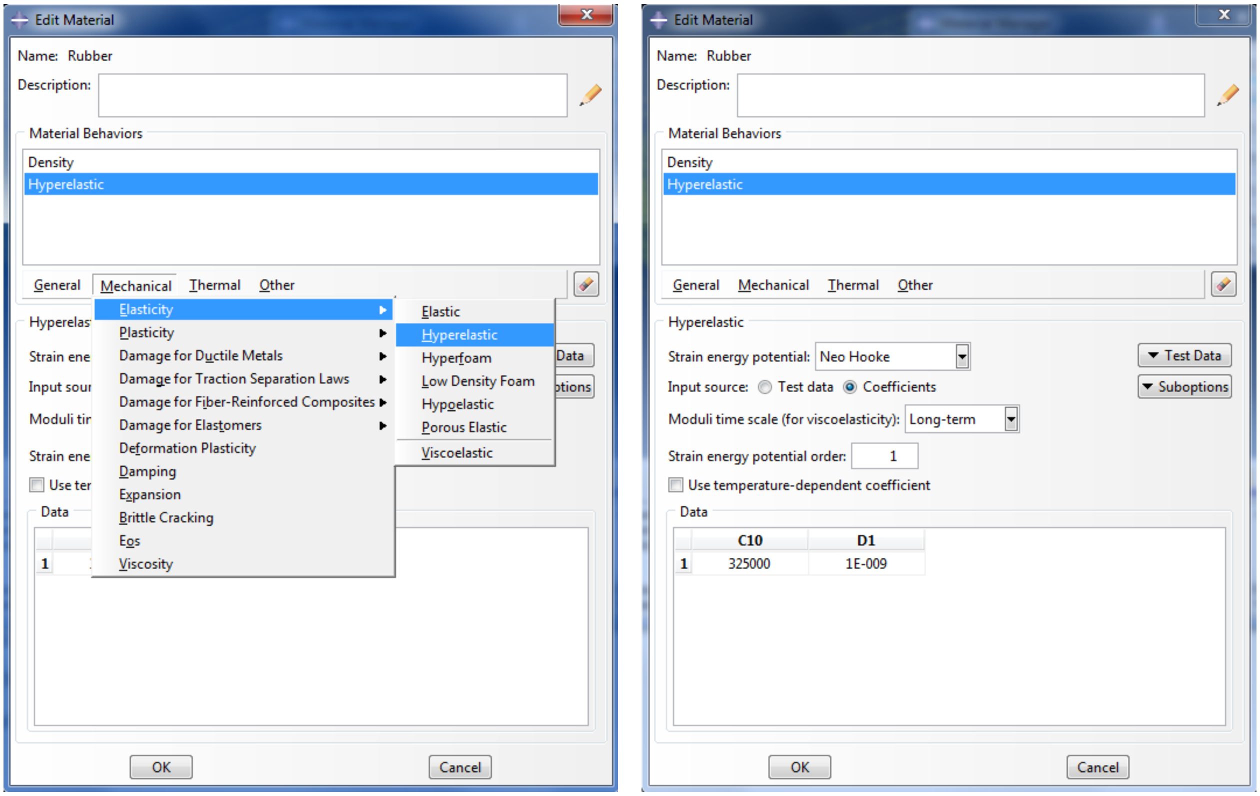


Figure 1.5: Finite element model of a low damping rubber bearing

Steel is modeled as a linear elastic material. Kinematic plastic or any other metal plasticity material model can be used if yielding in the internal steel shims is expected. Rubber can be modeled either as hyperelastic material or a viscoelastic material. Viscoelastic model can be used if experimental data on stress-strain, strain-rate, creep, and stress relaxation test is available. If only stress-strain data are available from an experiment, a hyperelastic material model is recommended. The property data form of rubber defined using a hyperelastic material is shown in Figure 1.6.

Hyperelastic material models require experimental data to determine the unknown parameters required to model the rubber. If the initial shear modulus of the rubber is known, Neo-Hookean model (Rivlin, 1948) is an appropriate model for tensile shear strains of up to 40%, in which case no experimental data is required. A high value is assigned to the bulk modulus of rubber to account for its incompressibility. If a viscoelastic material model is used for rubber, LS-DYNA requires the user to input a short-term and long-term shear modulus; in ABAQUS it can modeled in the frequency domain using complex modulus or in the time domain by constructing a Prony series.

The behavior of rubber in high shape factor bearings differs from the experimental behavior observed from rubber coupon tests. When an elastomeric bearing is subjected to vertical tension, cavities form inside the volume of rubber: cavitation. Cavitation is followed by the substantial reduction in the vertical stiffness. Although rubber damage models are available in ABAQUS (e.g., Mullins damage model), the cavitation phenomenon in elastomeric bearings is different and cannot be captured using the Mullins damage model. Cavitation in elastomeric bearings cannot be captured by ABAQUS, LS-DYNA, or ANSYS using continuum modeling approach at this time.



a) Selection of a mechanical properties **b) Definition of a Neo-Hookean material**

Figure 1.6: Properties definition of rubber material in ABAQUS

Convergence is often an issue with the use of hyperelastic material models due to its highly nonlinear characteristics. The single-parameter Neo-Hookean hyperelastic material, although easy to use, is not suitable for very large deformation analysis in which strain values exceeds 40%, which may be the case in the analysis of seismic isolation bearings. Other hyperelastic models (e.g., Yeoh and Ogden) using a larger number of parameters provide better numerical stability, but they require experimental data to determine the parameters.

1.5.2 Discrete Modeling

1.5.2.1 General

FEA software programs also provide option to create discrete model of isolators. Although the continuum approach modeling of isolators among different FEA software programs is similar, the modeling options for creating discrete models of isolators vary across platforms. Some of the software programs provide a direct option to model a seismic isolator based on geometrical and material properties of isolator, whereas others use different techniques to create a two-node, twelve degree-of-freedom element with six principal directions. The discrete modeling techniques in ABAQUS and LS-DYNA are discussed here. ANSYS does not provide a direct option for discrete modeling of isolators, but link and spring elements can be used to model an isolator.

1.5.2.2 Discrete Modeling in ABAQUS

ABAQUS provides the option to use a "connector" element to create discrete models of isolators. The connector element in ABAQUS is similar to Link/support element in SAP2000. ABAQUS provides a comprehensive list of "connector" elements that can be used to model an elastic spring, a dashpot, friction, plasticity, and damage. Different directions between two nodes can be coupled, uncoupled or a combination of both. An illustration of connector behaviors in ABAQUS is shown in Figure 1.7.

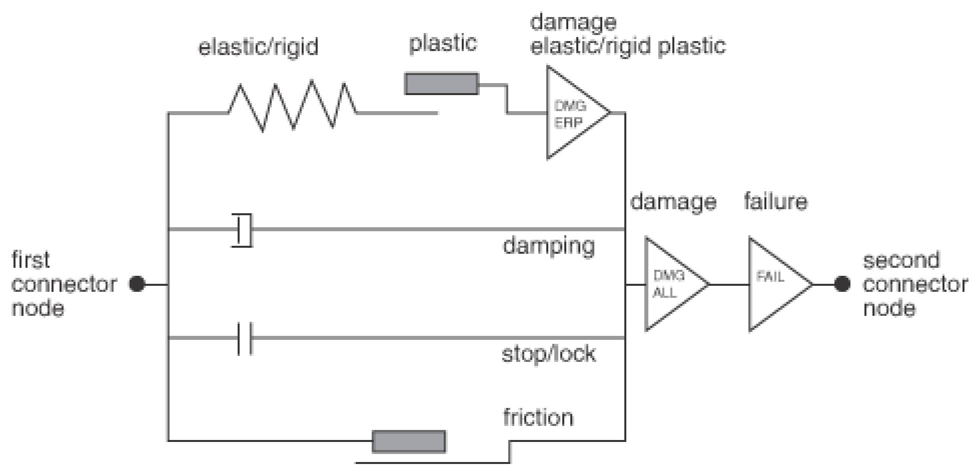
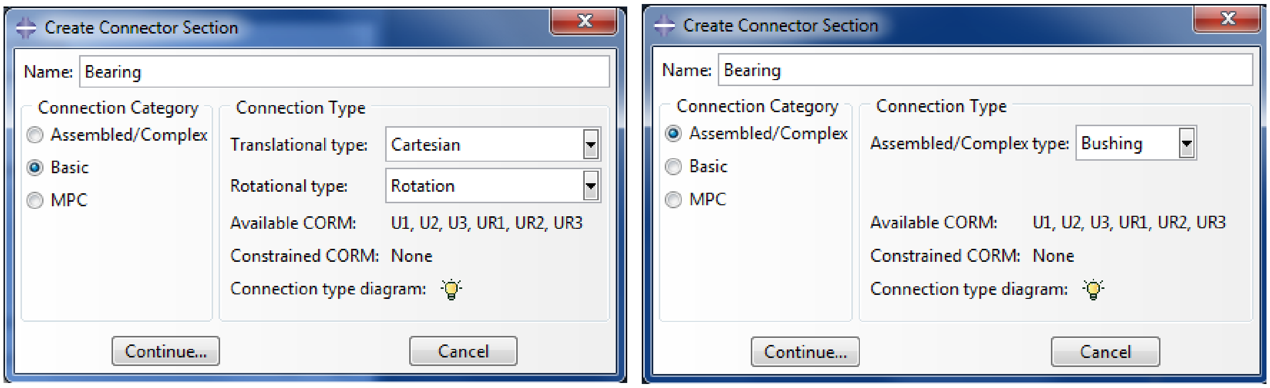


Figure 1.7: Conceptual illustration of connector behaviors (Systèmes, 2010b)

The first step in developing a connector is to define a connection type that represents the physical model of the isolator. There are two ways to create two-node, twelve degree-of-freedom discreet element that is characterized by six local directions: 1) a basic category connection with a translational connection type assigned to Cartesian and rotational connection type assigned to Rotation, or 2) an assembled/complex category Bushing connection. Both options are shown in Figure 1.8.



a) Basic connection **b) Assembled/complex connection**

Figure 1.8: Type of connectors used for seismic isolators

Once the connection type is defined, connection behavior can be defined in each of the six local directions for the seismic isolators. ABAQUS provides option of using an isotropic or kinematic hardening model. A direct option to use the Bouc-wen model extended by Nagarajaiah *et al.* (1991) for seismic isolators is not available. Figure 1.9 shows the data form for a connector. Additional information on use of connector elements in ABAQUS is provided in Section 28 of ABAQUS Analysis User's Manual (Systèmes, 2010b) and Section 15.7, 15.8 and 15.17 of ABAQUS/CAE User's Manual (Systèmes, 2010c).

The use of a discrete model of isolators using connector elements in ABAQUS reduces the computational effort drastically from that associated with a continuum model, and most of the nonlinear behaviors can be captured. However, modeling of isolators using connector elements in ABAQUS is involved by comparison with the discrete models available in structural analysis software programs such as SAP2000 and OpenSees.

ABAQUS also allows the user to define a model, which is not available in ABAQUS, through user subroutine code and then to integrate (link) it to ABAQUS for analysis. The two-node discrete model of elastomeric bearings can be implemented in ABAQUS by creating new subroutines called User Elements (UELs). The computational efficiency can be increased significantly, and it can capture all of the behaviors of seismic isolators observed experimentally that are defined by the user in the subroutine. (Two new UELs have been created in ABAQUS by Kumar *et al.* (2013a) for discrete models of LDR and LR bearings, respectively. These elements take only material and geometrical property of an isolator as input arguments.)

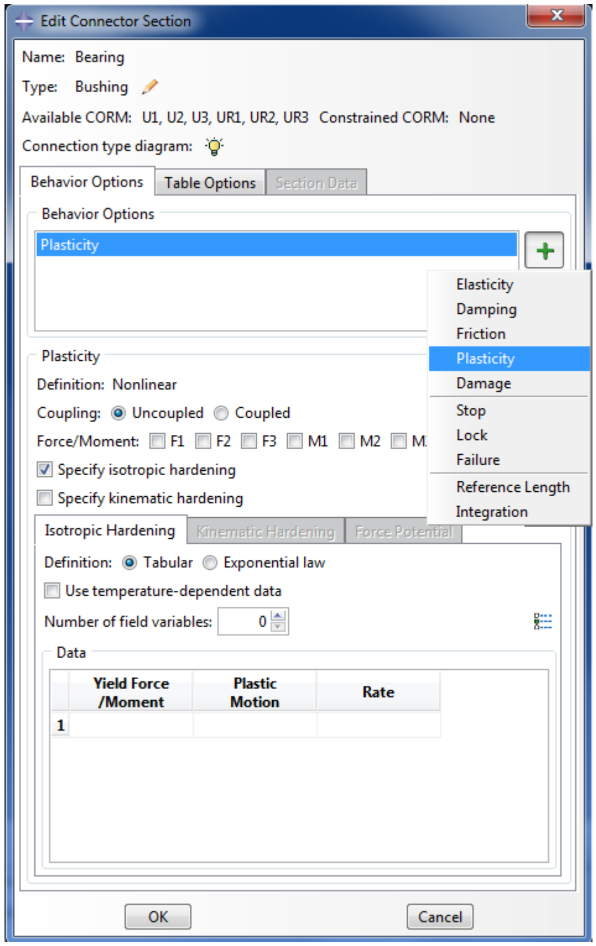


Figure 1.9: Definition of connector's behavior

1.5.2.3 Discrete Modeling in LS-DYNA

LS-DYNA provides a direct option to model elastomeric bearing through a material model option *MAT_SEISMIC_ISOLATOR. The corresponding element and section is created using options *ELEMENT_BEAM and *SECTION_BEAM, respectively. ELFORM is set to 6 (discrete beam), and the local axes of the isolator is defined in *SECTION_BEAM option. This material can be used to model elastomeric bearings, flat slider bearings, single FP bearings, double FP bearings and XY-FP bearings. Triple FP bearings can be modeled using three of such elements in series based on the procedure described in Fenz and Constantinou (2008). Behavior in two horizontal (shear) directions is similar to SAP2000 and OpenSees, which is based on the model proposed by Park *et al.* (1986) and extended for seismic isolators by Nagarajaiah *et al.* (1989). The vertical stiffness for all types of isolators is linear elastic, with the option to provide different values in compression and tension. The default value of tensile stiffness assigned to sliding isolators is zero. The element has no rotational or torsional stiffness and a pinned joint is assumed. However, if required, moments can be calculated according to the vertical load times the lateral displacement of the isolator by assigning the moment factors in the *MAT definition. Additional details on modeling a seismic isolator using *MAT_SEISMIC_ISOLATOR material model is provided in *MAT_197 of LS-DYNA Keyword User's Manual (LSTC, 2012b).

1.6 OpenSees

OpenSees provides more flexibility in modeling of isolators because of its code-based approach to construct the finite element model of the structure. Currently OpenSees has one element to model elastomeric bearings (LDR and LR), and four elements to model different types of friction-based isolators. These OpenSees elements can be used for two-dimensional or three-dimensional model of isolators. The three-dimensional representation of an isolator and associated degrees of freedom are shown in Figure 1.10.

All elements use the model proposed by Park *et al.* (1986) as extended for seismic isolation bearings by Nagarajaiah *et al.* (1991) to capture coupled behavior in two horizontal shear directions. The elastomeric bearing element (Schellenberg, 2006a) uses the mechanical properties of elastomeric bearing as input parameters to describe the force-deformation relationships. A user can assign any material model available in the OpenSees material library in the vertical (axial) direction. A linear elastic material with vertical stiffness calculated from a two-spring model (Koh and Kelly, 1988; Warn *et al.*, 2007) or a bilinear model (Constantinou *et al.*, 2007) is usually used in the vertical direction. This element cannot capture coupling of horizontal and vertical motion, cavitation in tension, strength reduction in cyclic tensile loading, heating of the lead core (in LD bearings) under large cyclic displacements, and variations in the critical buckling load of the bearing with displacement. (Two new elements have been created by Kumar *et al.* (2013a) for LDR and LR bearing, which incorporates these features in the element. Moreover, the new elements take only the geometrical and material properties of elastomeric bearing as input arguments and the appropriate mechanical properties are calculated by the element.) Four elements are available in OpenSees to model friction-based isolators. Table 1.1 summarizes the options.

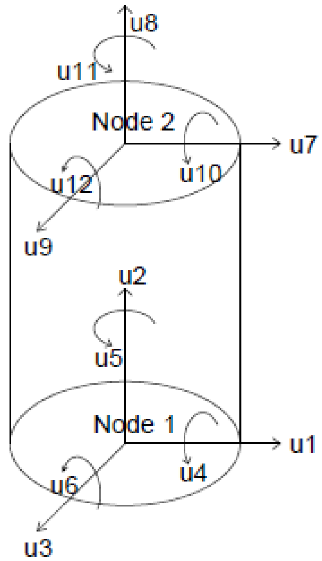


Figure 1.10: OpenSees isolator model

Table 1.1: Properties of the element used for friction-based isolator

Properties		flatSliderBearing	singleFPBearing	TFP	TripleFriction Pendulum
Coupled bidirectional motion		Yes	Yes	Yes	Yes
Friction variation	Velocity	No	No	No	Yes
	Pressure	No	No	No	Yes
	Temperature	No	No	No	No

The construction of elements for flat sliding bearings (flatSliderBearing) and the single Friction Pendulum (FP) bearing (singleFPBearing) is similar to the element used for an elastomeric bearing (Schellenberg, 2006b; Schellenberg, 2006c). A friction model must be chosen from the OpenSees library and assigned to these two elements. The TFP element (Becker and Mahin, 2012) takes geometrical and material properties of triple FP bearing as input arguments and calculates the mechanical properties. It modifies the material model provided by the user in vertical direction for the compression-only behavior of the triple FP bearing. The TripleFrictionPendulum element (Dao *et al.*, 2013) also takes geometrical and material properties of the triple FP bearing as input arguments. This element can simulate the pressure and velocity dependence of friction. Work is under progress at University at Buffalo to include the temperature dependence of friction in an OpenSees element (Kumar *et al.*, 2013b).

1.7 Summary

All of the available software programs discussed here are capable of modeling seismic isolators with varying degrees of sophistication. Although, the effort required also depends on user's familiarity with the particular software program, some programs provide a direct option to model an isolator element based on their material, geometrical and mechanical properties, whereas in others, several elements need to be combined to produce isolator-like behavior, or a continuum-based approach needs to be used. The special-purpose software programs used for structural analysis (e.g., SAP2000, OpenSees, PERFORM-3D, and 3D-BASIS) enable modeling of simple isolator behaviors, but complex behaviors (e.g., cavitation, interaction between axial compression and shear stiffness, strength degradation) cannot be captured, except in the open-source code, OpenSees. The general-purpose software programs (LS-DYNA, ABAQUS, and ANSYS) have the capability to model complex isolator behaviors using either discrete or continuum based approaches. Among the three general-purpose FEA programs discussed here, only LS-DYNA provides a direct option to model an isolator based on its material and geometrical properties. Isolators modeled using continuum-based approach may not be amenable for analysis of large models. The continuum approach is recommended when the behavior of an individual isolator is to be studied. For analysis of large base-isolated structure, discrete model of isolators should be used.

Table 1.2 and Table 1.3 present the capability of the seven programs summarized in this section to model different types of isolators and their behaviors.

Table 1.2: Modeling of elastomeric seismic isolators and software programs

Properties	3DBASIS	SAP2000	PERFORM3D	LSDYNA	ABAQUS	OpenSees	New Elements[1]
Coupled horizontal directions	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Coupled horizontal and vertical directions	No	No	No	No	No	No	Yes
Different tensile and compressive stiffness	No	No	Yes	Yes	Yes	Yes	Yes
Nonlinear tensile behavior	No	No	No	No	Yes	Yes	Yes
Cavitation and post-cavitation	No	No	No	No	No	No	Yes
Nonlinear compressive behavior	No	No	No	No	Yes	Yes	Yes
Varying buckling capacity	No	No	No	No	No	No	Yes
Heating of lead core	No	No	No	No	No	No	Yes

1. New elements created for LDR and LR bearings in OpenSees and ABAQUS at University at Buffalo with funding from USNRC

Table 1.3: Modeling of friction-based isolators and software programs

Properties	3DBASIS	PERFORM3D	ABAQUS	LSDYNA	SAP2000	OpenSees	New Elements[1]
Coupled horizontal directions	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Different tensile and compressive stiffness	No	Yes	Yes	Yes	Yes	Yes	Yes
Modeling of XY-FP bearing	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Modeling of Triple FP bearing	Yes	No	No	Yes	Yes	Yes	Yes
Velocity dependence of friction	No	Yes	Yes	Yes	Yes	Yes	Yes
Pressure dependence of friction	No	No	No	No	No	Yes	Yes
Temperature dependence of friction	No	No	No	No	No	No	Yes

1. New elements created for FP bearings in OpenSees at University at Buffalo with funding from USNRC

The difficulty of modeling seismic isolators in different software programs and the computational efficiency of different software programs in analysis of seismically isolated structures is presented in Figure 1.11 and Figure 1.12. The software programs are rated at 1 to10 scale. Please note that the figures present a subjective opinion of the author and is not based on any controlled research, as such.

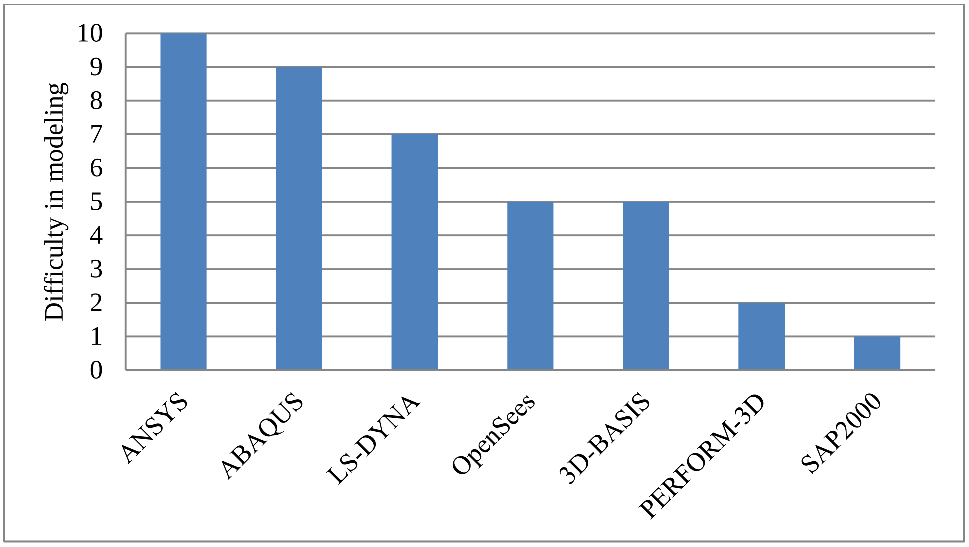


Figure 1.11: Difficulty rating of modeling of seismic isolators in software programs

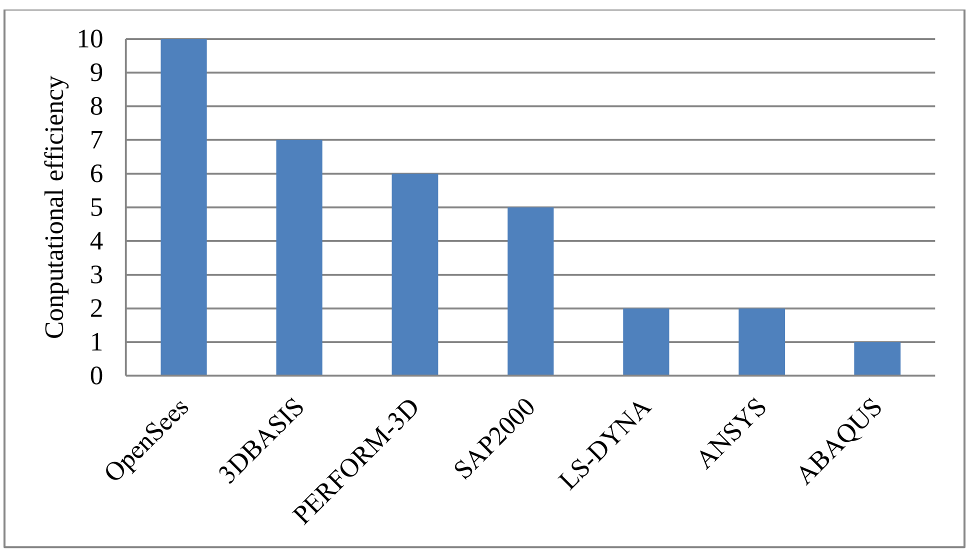


Figure 1.12: Computational efficiency of software programs in analysis of seismic isolators

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