SECTION 1

SEISMIC ANALYSIS OF THREE-STORY FRAME

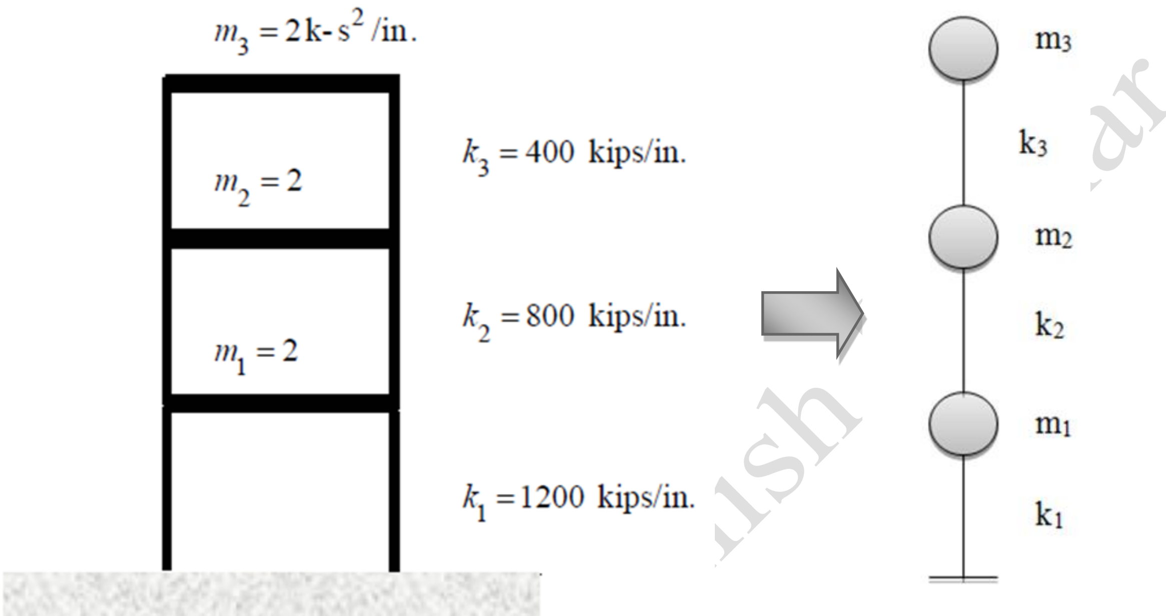

Figure 1: Three-story frame structure and equivalent mathematical model

1.1 Eigenvalue Analysis

The aim of this eigenvalue analysis is to determine the mode shapes and frequencies of the frame shown above. Given frame is idealized as shear building with no rotational and axial degrees of freedom and has only three translational degrees of freedom in horizontal directions at each floor. A simplified model of given frame is shown as stick-mass model in Figure 1.

Equations of motions are written for each floor separately as:

$$m_3\ddot{u}_3 + k_3\left(u_3 - u_2\right) = 0 \tag{1.1}$$

$$m_2\ddot{u}_2 + k_3\left(u_2 - u_3\right) + k_2\left(u_2 - u_1\right) = 0 \tag{1.2}$$

$$m_1\ddot{u}_1 + k_1 u_1 + k_2\left(u_1 - u_2\right) = 0 \tag{1.3}$$

Writing the above set of equations in matrix form:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = 0 \tag{1.4}$$

Or,

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \tag{1.5}$$

Now, if the displacement vector **u** is decomposed into mode-shapes and time variation, it can be written as:

$$\{u\} = q_n(t)\phi_n \tag{1.6}$$

Where, ϕ_n is mode-shape vector for nth mode and $q_n(t)$ is the time variation that can be described as:

$$q_n(t) = A_n \cos\omega_n t + B_n \sin\omega_n t \tag{1.7}$$

ω_n is the natural frequency of nth mode.

Substituting back the values in Eq. (1.5), we get our eigenvalue problem as:

$$K\phi_n = \omega_n^{\ 2} M\phi_n \tag{1.8}$$

Multiplying both sides by inverse of mass matrix we obtain:

$$M^{-1}K\phi_n = \omega_n^{\ 2}\phi_n \tag{1.9}$$

Which is of the form $Av = \lambda v$, with $A = M^{-1}K$, and $\lambda = \omega_n^{\ 2}$.

The eigenvalue problem was solved using matlab code to obtain frequencies and mode shapes. Obtained frequencies and modal co-ordinates are presented in Table 1 and mode shapes are plotted in Figure 2.

Table 1: Frequencies and mode shapes

Modes	1	2	3
Angular frequency(rad/sec)	9.12	21.42	35.47
Time period(sec)	0.689	0.291	0.177
1^{st} floor	1	1	1
2^{nd} floor	2.292	1.353	-0.645
3^{rd} floor	3.923	-1.045	0.122

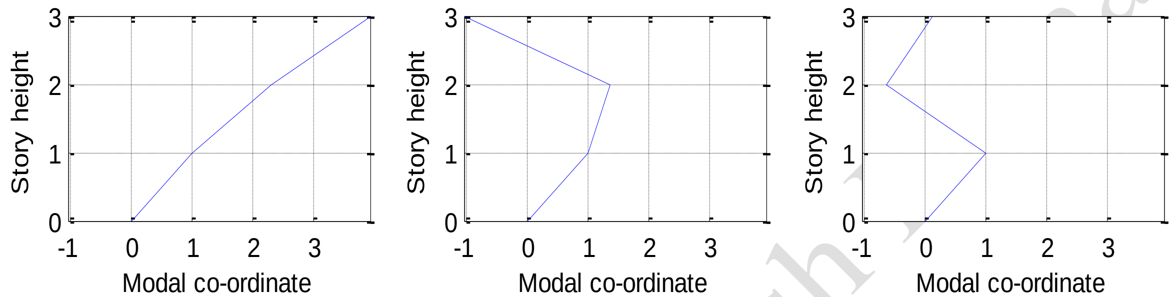

Figure 2: Mode-shapes

1.2 Equation of motion

As derived in Section 1.1, equation of motion is written as:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{u}_1 \\ \ddot{u}_2 \\ \ddot{u}_3 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = 0 \tag{1.10}$$

1.3 Response of the structure

$$M = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} k - s^2 / in$$

$$K = \begin{bmatrix} 2000 & -800 & 0 \\ -800 & 1200 & -400 \\ 0 & -400 & 400 \end{bmatrix} kips / in$$

$$\omega_n = \begin{Bmatrix} 9.12 \\ 21.42 \\ 35.47 \end{Bmatrix} rad/\sec, \Phi = \begin{bmatrix} 1 & 1 & 1 \\ 2.292 & 1.353 & -0.645 \\ 3.923 & -1.045 & 0.122 \end{bmatrix}$$

Modal properties obtained above are used to estimate the response of the structure. Spectral co-ordinates are obtained from design spectrum constructed using NEHRP recommended provisions (Council, 2009).

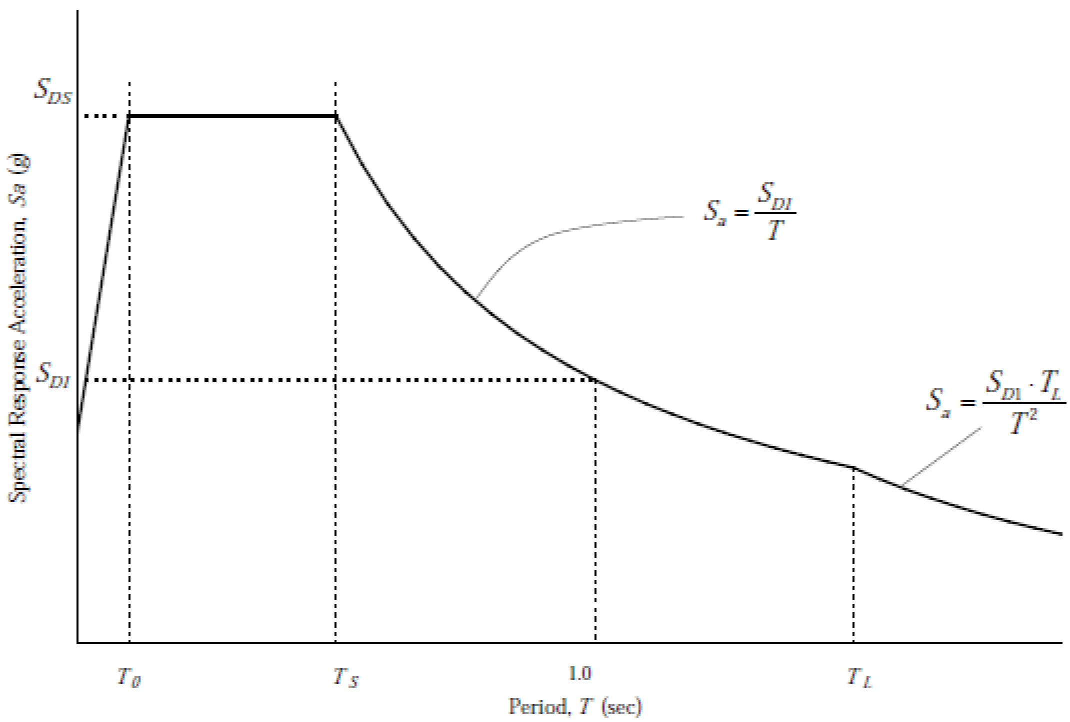

Figure 3: Design spectrum from NEHRP recommended provisions (Council, 2009)

As specified in the problem statement, $S_{DS} = 1.2g$, $S_{D1} = 0.4g$.

$$T_0 = 0.2\frac{S_{D1}}{S_{DS}} = 0.6\sec$$

$$T_s = \frac{S_{D1}}{S_{DS}} = 3\sec$$

Values of spectral acceleration for different range of time periods are given as:

For $T < T_0$

$$S_a = S_{DS}\left(0.4 + 0.6\frac{T}{T_0}\right) \quad (1.11)$$

For $T_0 < T < T_s$

$$S_a = S_{DS} \quad (1.12)$$

For $T_s < T < T_L$

$$S_a = \frac{S_{D1}}{T} \quad (1.13)$$

Using equations presented above, values of spectral acceleration for three modes are obtained and presented in

Table 2: Spectral response for three modes

Mode	Angular frequency (rad/sec)	Time period (sec)	Acceleration (g)	Displacement (inch)
1	9.12	0.689	1.200g	5.57
2	21.42	0.293	0.832g	0.70
3	35.47	0.177	0.692g	0.21

Modal participation factor is given by:

$$\Gamma_n = \frac{\{\phi_n\}[M]\{r\}}{\{\phi_n\}[M]\{\phi_n\}^T} \quad (1.14)$$

Once modal participation factor for each mode is obtained, maximum displacement, shear force, and overturning moment are given by:

Maximum displacement: $u_{n,\max} = \phi_n \Gamma_n D_{n,\max}$ (1.15)

Story intertia forces: $F_{s,n} = [M]\{\phi_n\}\Gamma_n S_{a,n}$ (1.16)

$$\text{Base shear: } V_{bn,\max} = \Gamma_n \sum_{j=1}^{N} m_j \phi_{jn} A_{n,\max} = \frac{\left(\sum_{j=1}^{N} m_j \phi_{jn}\right)^2}{\sum_{j=1}^{N} m_j \phi^2_{jn}} A_{n,\max} = M^*_n A_{n,\max} \quad (1.17)$$

$$\text{Overturning moment: } M_{bn,\max} = \frac{\sum_{j=1}^{N} h_j m_j \phi_{jn}}{\sum_{j=1}^{N} m_j \phi_{jn}} V_{bn,\max} = h^*_n V_{bn,\max} \quad (1.18)$$

Again, Matlab was used to calculate the maximum response in each mode, and the values are presented in Table 3.

Table 3: Modal response of structure

Mode		1	2	3
Period (sec)		0.689	0.293	0.177
Modal participation factor		0.333	0.333	0.333
Displacement (inch)	Floor 1	1.857	0.233	0.070
	Floor 2	4.256	0.316	-0.045
	Floor 3	7.284	-0.244	0.009
Inertia forces (kips)	Floor 1	309	214	178
	Floor 2	708	290	-115
	Floor 3	1213	-224	22
Story shear (kips)	Floor 1	2230	280	85
	Floor 2	1921	66	-93
	Floor 3	1213	-224	22
Base shear (kips)		2230	302	88
Overturning moment (kips-in)		643740	14660	1620

Approximate total maxima of each response quantity are obtained using SRSS combination rule and values are presented in Table 4.

Table 4: Absolute maximum response

Response parameter		value
Displacement (inch)	Floor 1	1.87
	Floor 2	4.27
	Floor 3	7.28
Inertia forces (kips)	Floor 1	416
	Floor 2	774
	Floor 3	1234
Story shear (kips)	Floor 1	2249
	Floor 2	1925
	Floor 3	1234
Base shear (kips)		2252
Overturning moment (kips-in)		643909

1.4 SAP2000 Model

Simplified model of three-story building was created in SAP2000. As width of frame was not provided, an arbitrary width was chosen. Width of frame does not affect the dynamic properties of frame, if beams are very stiff compared to columns (Chopra, 2007). Rigid beam simulates the actual behavior of three-dimensional shear building in which rigidity is provided by stiff slab at each floor. A general frame section with high magnitude of geometrical parameters was used for beam in SAP model to provide the required rigidity.

Column sizes were proportioned using stiffness of each story provided in the problem statement. Section designer tool in SAP was used to create square steel sections for columns. Mass was distributed to nodes at each floor. Model prepared in SAP2000 is shown in Figure 4. Zero material density was assigned to ignore the self-weight of the structure.

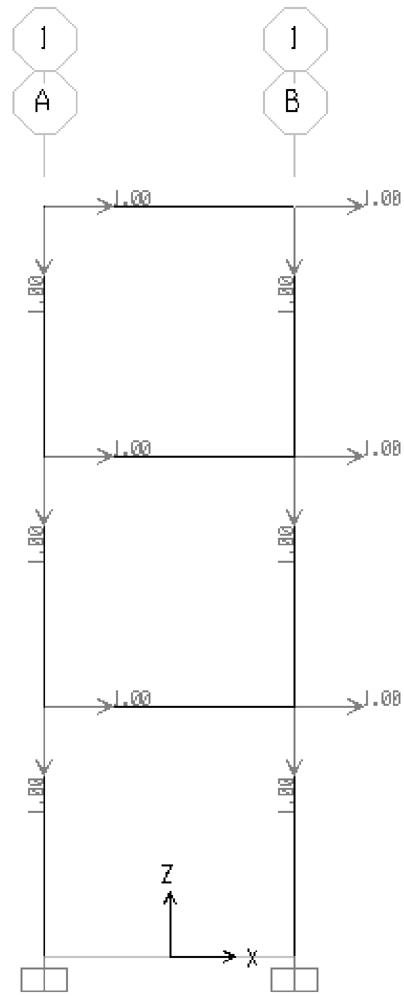

Figure 4: Model of three-story frame prepared in SAP2000

Modal analysis was performed, and obtained modal properties are compared with analytical solution in Table 5.

Table 5: Comparison of modal response

Modes	1		2		3	
	Analytical	SAP	Analytical	SAP	Analytical	SAP
Angular frequency(rad/sec)	9.12		21.42		35.47	
Time period(sec)	0.689	0.700	0.291	0.298	0.177	0.180
1st floor	1	1	1	1	1	1
2nd floor	2.292	2.283	1.353	1.340	-0.645	-0.649
3rd floor	3.923	3.887	-1.045	-1.045	0.122	0.124

1.5 Ground Motions and Response Spectra

PEER ground motion database (2010) was used to extract 10 ground motions using criteria specified in the problem statement. Summary of selected ground motions in presented in Table 6.

Table 6: Summary of properties of selected horizontal records

NGA#	Event	Year	Station	Mag	Mechanism	Low.freq (Hz)
159	Imperial Valley-06	1979	Agrarias	6.53	Strike-Slip	0.06
165	Imperial Valley-06	1979	Chihuahua	6.53	Strike-Slip	0.06
179	Imperial Valley-06	1979	El Centro Array #4	6.53	Strike-Slip	0.12
183	Imperial Valley-06	1979	El Centro Array #8	6.53	Strike-Slip	0.12
821	Erzican- Turkey	1992	Erzincan	6.69	Strike-Slip	0.12
1111	Kobe- Japan	1995	Nishi-Akashi	6.9	Strike-Slip	0.12
1165	Kocaeli- Turkey	1999	Izmit	7.51	Strike-Slip	0.12
1176	Kocaeli- Turkey	1999	Yarimca	7.51	Strike-Slip	0.09
1605	Duzce- Turkey	1999	Duzce	7.14	Strike-Slip	0.1
1617	Duzce- Turkey	1999	Lamont 375	7.14	Strike-Slip	0.19

The PEER ground motion database provides three components of the ground motion along with the geometric mean of two horizontal components. Response spectra of geometric mean of horizontal components of 10 selected ground motion records have been plotted in Figure 5. The maximum, minimum, and median response spectrum of selected ground motions are plotted in Figure 6.

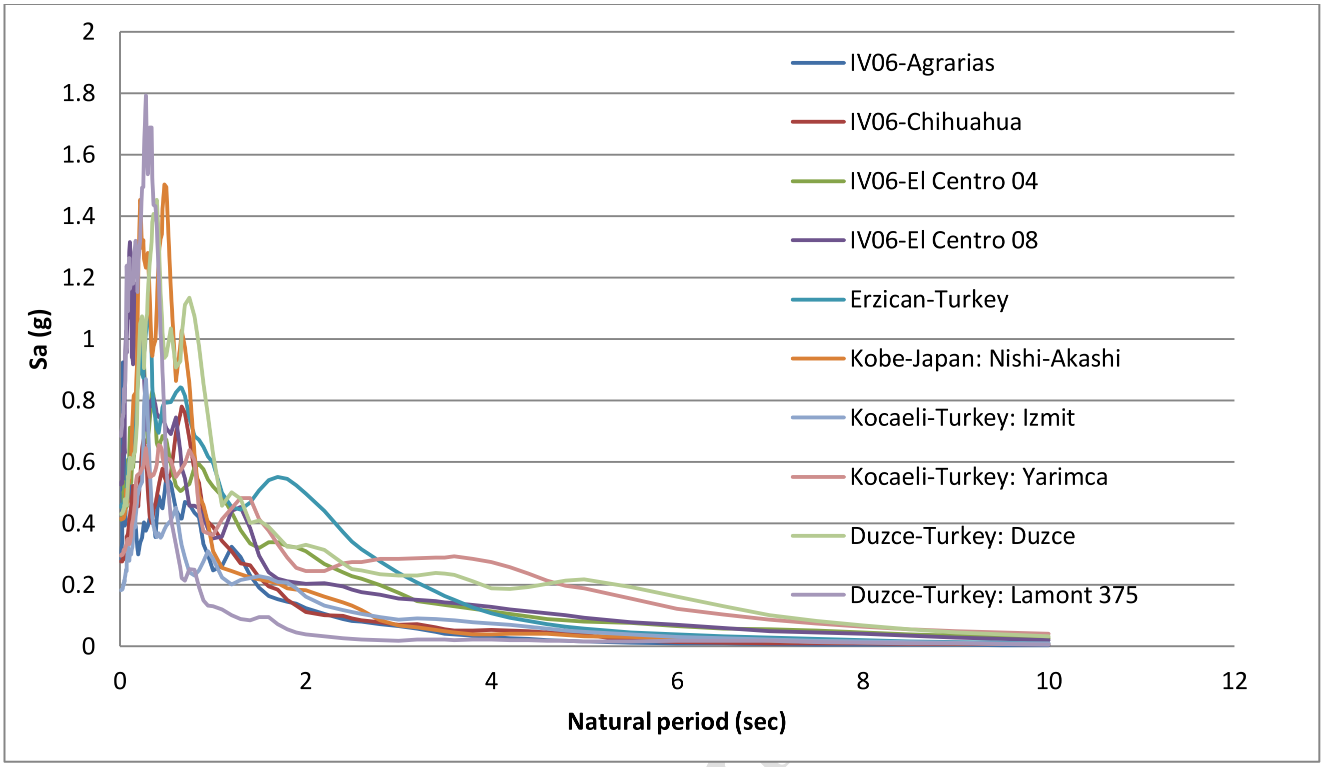

Figure 5: Response spectra of selected ground motions

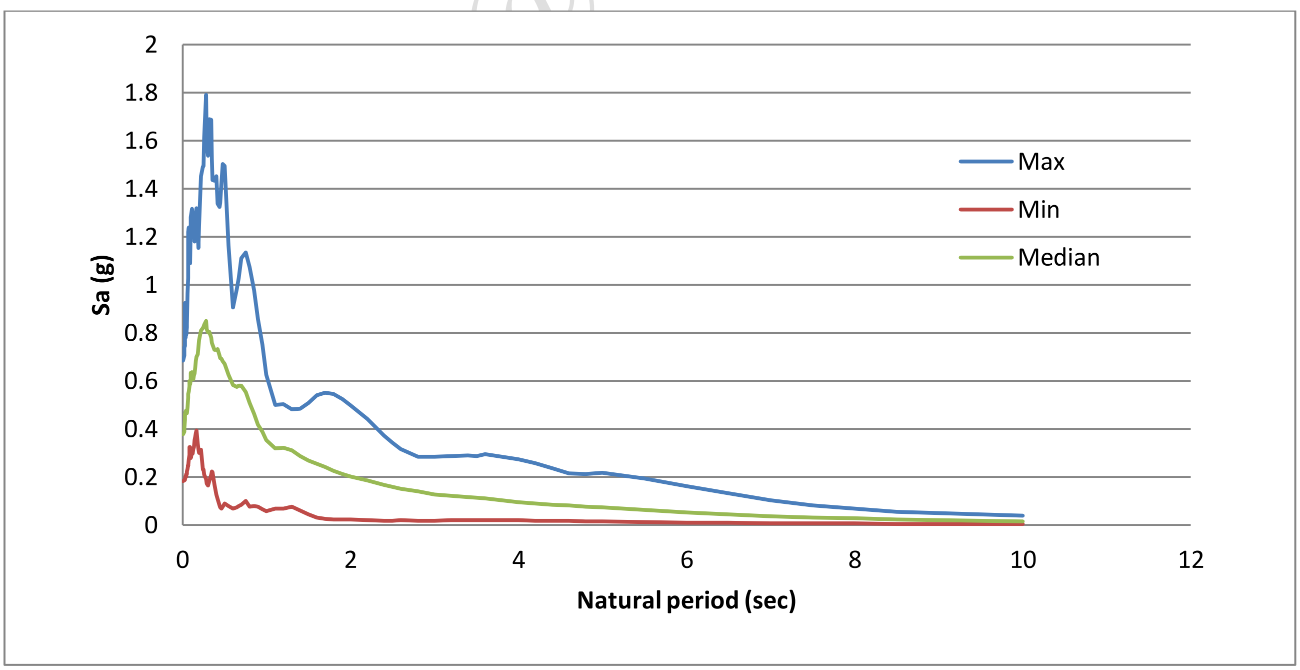

Figure 6: Variation of response spectra of selected ground motions

Maximum, minimum, and median story-drifts and peak floor accelerations from SAP response history analyses and are presented in Figure 7 and Figure 8.

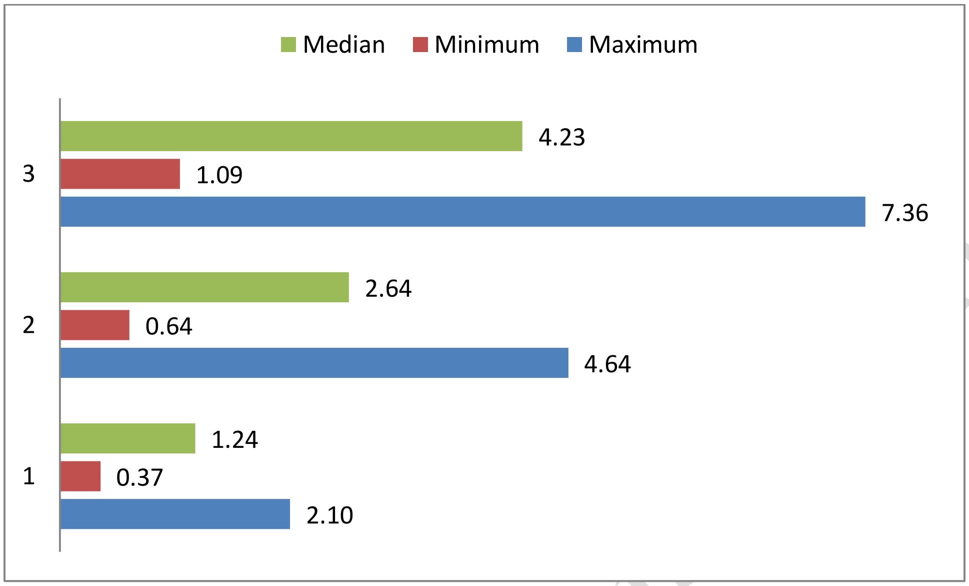

Figure 7: Story-drift distributions along the height of frame (values in inch)

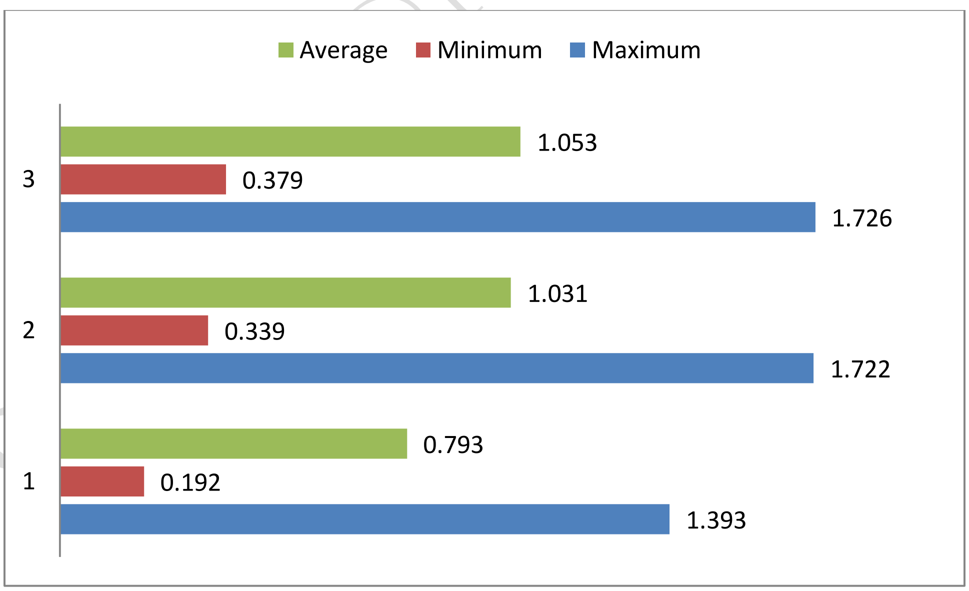

Figure 8: Acceleration distributions along the height of frame (values in g)

Maximum peak floor acceleration at 3rd floor was produced by Kobe earthquake (NGA# 1111). Acceleration history was obtained at the 3rd floor from SAP2000 using plot functions tool and imported in SeismoSignal to generate the floor response spectrum. Zero-period spectral acceleration was matched with peak floor acceleration as a check. Floor response spectrum is presented in Figure 9.

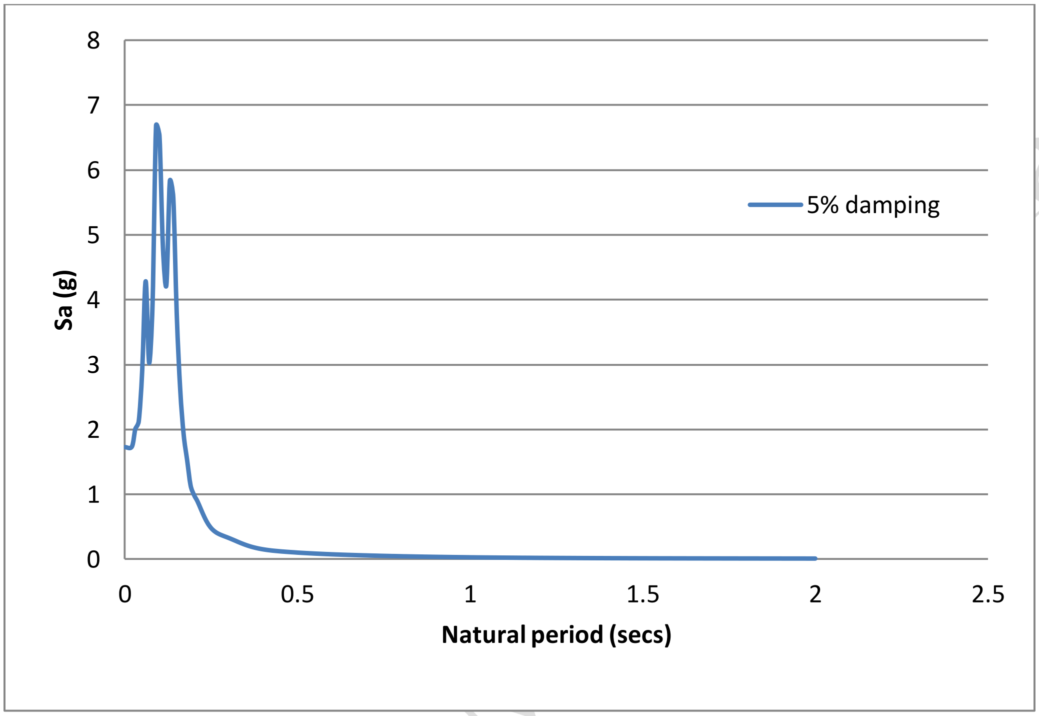

Figure 9: Floor response spectrum for 3rd floor

It can be seen from the spectrum that equipment on 3rd floor might experience acceleration as high as 7.0g. Distribution of spectrum suggests that stiff, or high frequency components would be more affected compared to low frequency components. A good strategy might be to use components with low frequencies with a certain cutoff frequency deduced from spectrum above.

Appendix A

```
%program for eigenanalysis, course: Advanced Earthquake Engineering
%written by: Manish Kumar, July, 2012
clc; clear

M=[2 0 0; 0 2 0; 0 0 2];
K=[2000 -800 0;
    -800 1200 -400; 0 -400 400];
H=[120 240 360]';
A=inv(M)*K;
[V,Dw] = eig(A);
[m,n] = size(Dw);
W = zeros(m,1);       % Preallocate matrix

for i = 1:m
    W(i,1)=sqrt(Dw(i,i));
end
for i=1:n
    c=V(1,i);
    for j=1:m
        V(j,i)=V(j,i)/c;
    end
end

for j=1:m
    subplot(1, ceil(m), j)
    plot([0;V(:,j)],linspace(0,m,m+1))
    xlabel('Modal co-ordinate'), ylabel('Story height')
    axis equal; axis([ -1.0453 3.9233 0 3]) %find a way to automize the
limits
    %PlotAxisAtOrigin(0,0)
    grid on
end
r=[1 1 1]';
A=386.4*[1.2 0.832 0.692]';
D=[5.57 0.7 0.21]';
T = zeros(m,1); %modal participation factor
for i=1:m
    T(i,1)=V(:,i)'*M*r/(V(:,i)'*M*V(:,i));
end

u = zeros(m,m); %story displacements
for i=1:m
    u(:,i)=V(:,i)*T(i,1)*D(i,1);
end

M_eq=zeros(m,1); %modal mass
for i=1:m
    M_eq(i,1)=(V(:,i)'*M*r)^2/(V(:,i)'*M*V(:,i));
end

Fs = zeros(m,m); %story intertia forces
for i=1:m
    Fs(:,i)=M*V(:,i)*T(i,1)*A(i,1);
```

```
end

H_eq=zeros(m,1); %modal height
for i=1:m
    H_eq(i,1)=V(:,i)'*M*H/(V(:,i)'*M*r);
end

Vbase=zeros(m,1); %base shear
for i=1:m
    Vbase(i,1)=M_eq(i,1)*A(i,1);
end

Mbase=zeros(m,1); %Overturning moment
for i=1:m
    Mbase(i,1)=Vbase(i,1)*H_eq(i,1);
end

Vstory=zeros(m,m); %story shear
for i=1:m
    s=0;
    for j=1:m
        s=s+Fs(m-j+1,i);
        Vstory(m-j+1,i)=s;
    end
end
```

References

Chopra, A. K. (2007). "Dynamics of structures : theory and applications to earthquake engineering." Prentice Hall, NJ, USA.

Council, B. S. S. (2009). "NEHRP recommended seismic provisions for new buildings and other structures." *Federal Emergency Management Agency*, , Report P-750, Washington, DC.