Elastomeric seismic isolation bearings

Modelling, analysis and design

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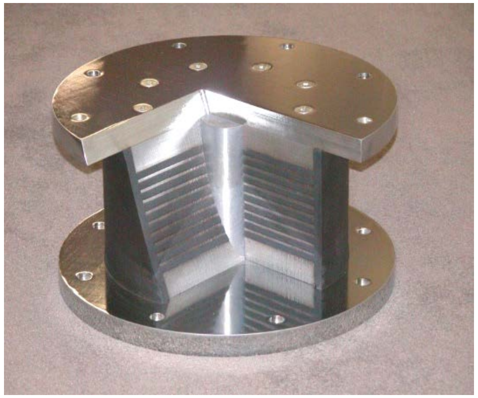
Outline

- Introduction
- Modeling techniques
  - Finite element
  - Discrete element
- Analysis methods
- Design procedures
- Advanced isolator models
- Contemporary softwares
- Conclusions

Seismic Isolator types

- Elastomeric/rubber bearings
  - Low damping rubber (LDR) bearings
  - Lead-rubber (LR) bearings
- Sliding bearings
  - Flat slider bearings
  - Friction pendulum (FP) bearings
    - Single FP bearings
    - Double FP bearings
    - Triple FP™ bearings
  - T/C friction isolator

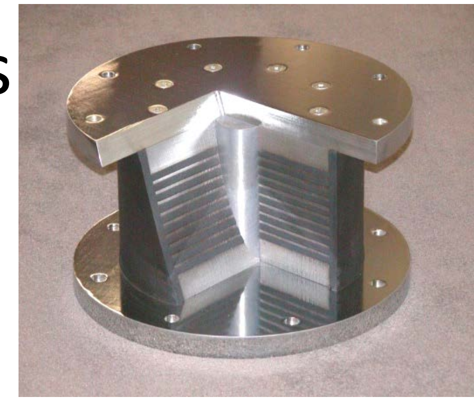


Internal section view of lead-rubber bearing

Friction (single pendulum) isolator

Elastomeric bearings

- Low damping rubber (LDR) bearings
- Lead-rubber (LR) bearings
  - LDR bearing + lead core
- High damping rubber bearings



Internal section view of lead-rubber bearing

- Rubber properties
  - Neoprene rubber
    - Synthetic rubber with carbon filler
  - Natural rubber
    - Naturally cured rubber

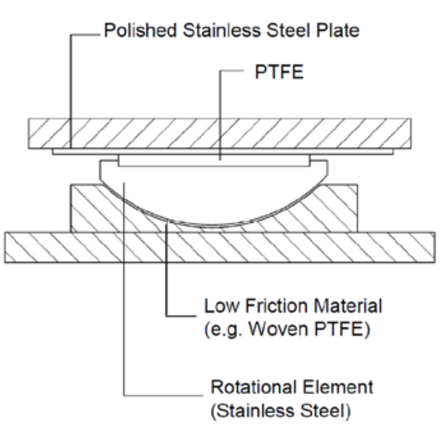
Erzurum Hospital, Turkey

Rubber bearing manufacturing

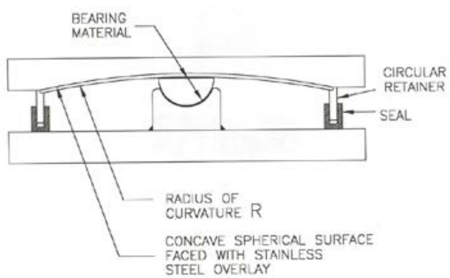
Process	**Description**
Mixing of rubber	Raw rubber, carbon black, sulfur and other additives are mixed
Sheeting (calendaring) of rubber	Rubber is cut into the desired shapes (circular, annular)
Cutting of rubber	Rubber is cut into the desired shapes (circular, annular)
Cutting of steel plate	End plates and shim plates of the required thickness are cut into desired shapes
Steel plate surface treatment	End plates and shim plates are sand-blasted
Application of adhesives	End plates and shim plates are coated with (proprietary) adhesives
Forming (lay-up) of bearing	End plates, shim plates and rubber sheets are assembled; cover rubber is placed on the outside of the bearing
Curing (vulcanization)	The formed bearing is set in a mold and cured under pressure and heat: rubber is vulcanized and bonded to the steel
Finishing	End plates are painted; lead-plug is inserted in for lead-rubber bearings

Courtesy of M. Constantinou, University at Buffalo

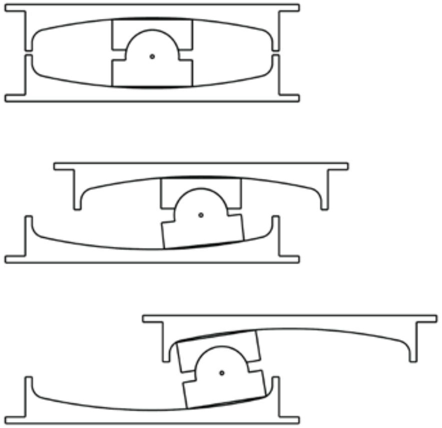
Sliding bearings



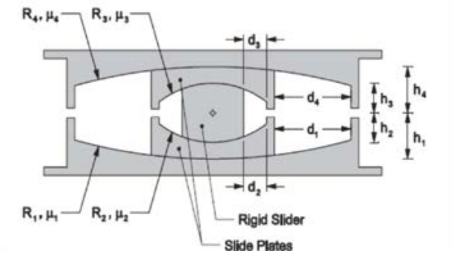
Flat-slider bearings

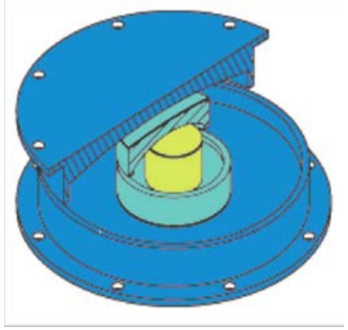


Single FP bearings



Double FP bearings

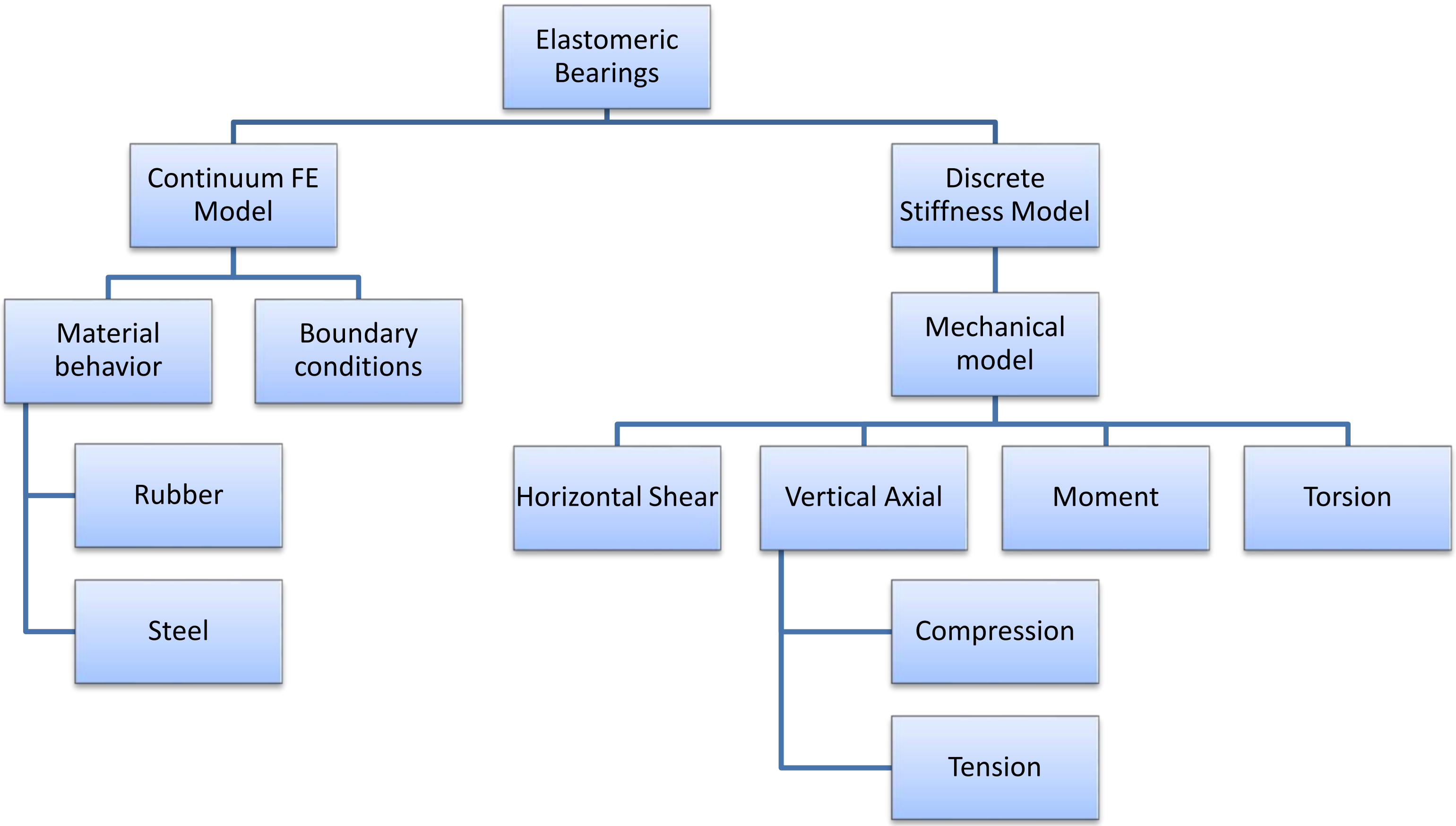




Triple FP™ bearings

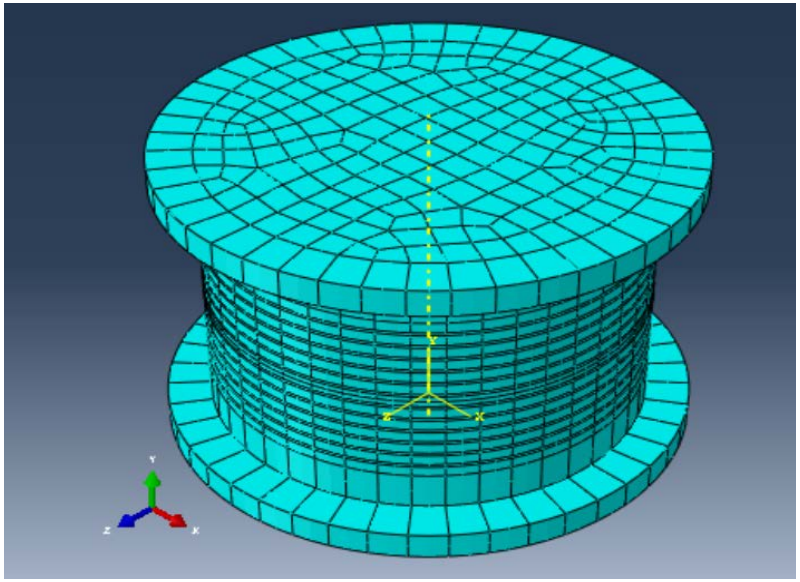
MODELING AND ANALYSIS

Modeling

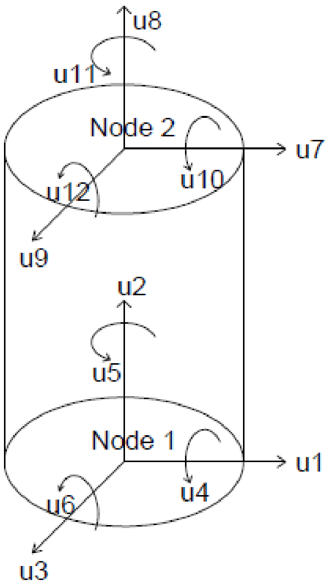

Elastomeric Bearings
Continuum FE Model
Discrete Stiffness Model
Material behavior
Boundary conditions
Mechanical model
Rubber
Steel
Horizontal Shear
Vertical Axial
Moment
Torsion
Compression
Tension

Modeling

- Continuum FE model is appropriate for studying component level response
  - Difficult to model and computational demanding
- Discrete model is required for base-isolated structure comprised of hundreds of such bearings
  - Simple to model and computationally efficient



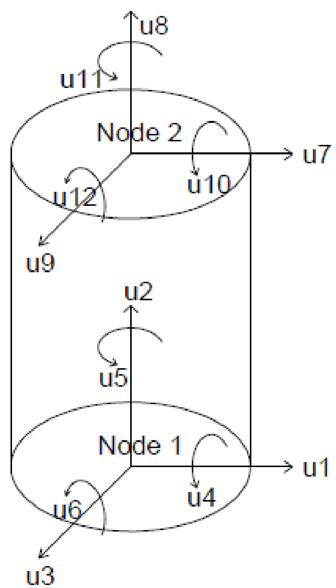
Continuum FE model



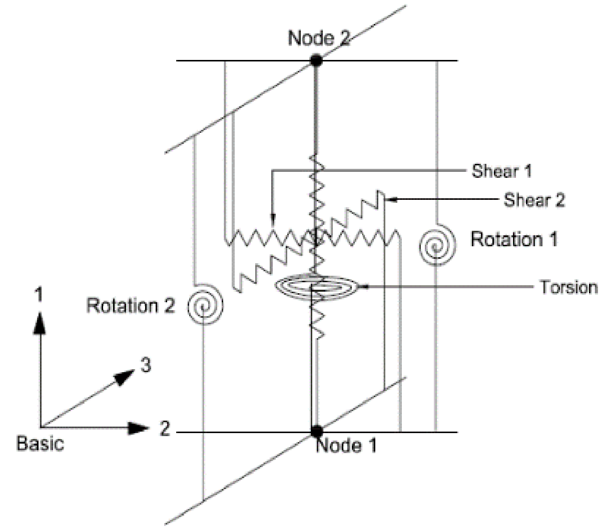
Discrete model

Discrete Model

- 2 Node, 12 DOFs
- Connected by 6 springs
  - Represents mechanical behavior in 6 directions



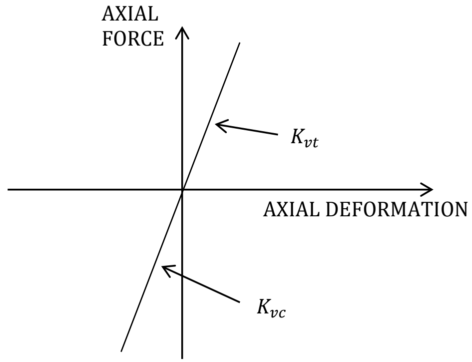
Physical model



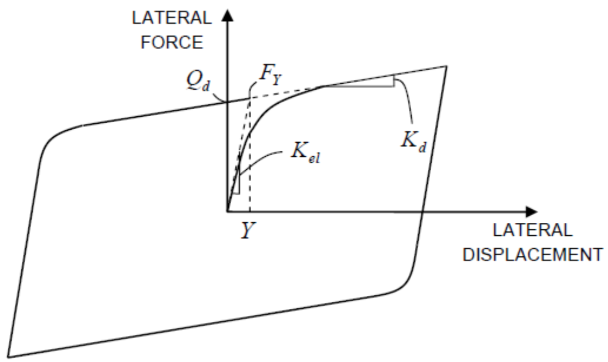
Discrete spring model

Modeling: state of practice

- Axial
  - Linear spring
- Shear (2 horizontal directions)
  - Bi-directional Bouc-Wen model
- Torsion
  - Linear elastic with stiffness = $\frac{GJ}{T_r}$
- Rotation (about 2 horizontal directions)
  - Linear elastic with stiffness = $\frac{IE_r}{T_r}$



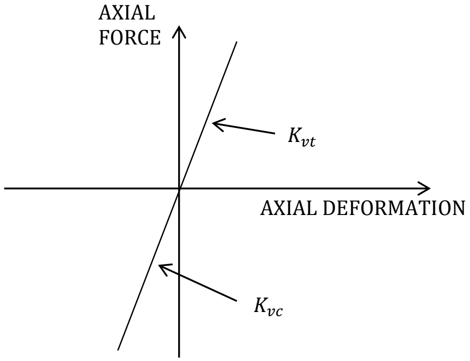
Idealized axial behavior



Idealized shear behavior

Axial behavior

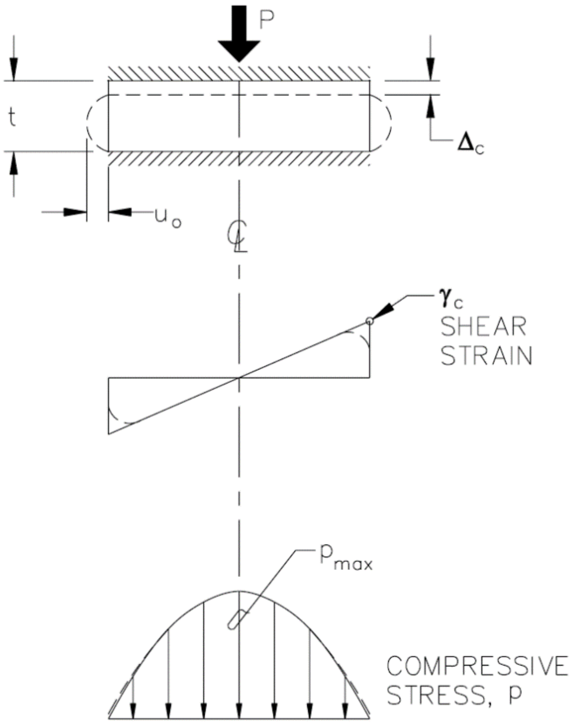
- Vertical axial stiffness is much greater than lateral stiffness
- Large vertical stiffness due to
  - Incompressibility of rubber
  - Lateral restrain provided by steel shims
- Implied infinite capacity under compression and tension
  - Allow simplified modeling



Idealized axial behavior

Axial stiffness

- Axial stiffness of multilayer bearings in compression
- $K_v = \dfrac{A}{\sum_i t_i\left[\frac{1}{E_{ci}}+\frac{4}{3K}\right]} = \dfrac{AE_c}{T_r}$
- $\sum_i t_i = T_r$ (total rubber layer thickness)
- $K = 2000$ MPa bulk modulus of rubber
- $E_{ci} = 6GS^2$ (compression modulus of a constrained rubber layer)
- Obtained using the "Pressure" solution of Constantinou et al. (1992).
- Compression Modulus
  $$E_c = \left(\frac{1}{6GS^2} + \frac{4}{3K}\right)^{-1}$$



Shape factor

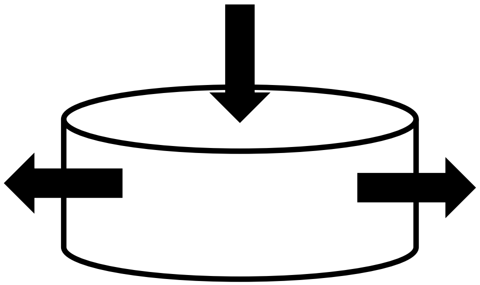
- Very important geometric parameter

$$S = \frac{Loaded\ area\ of\ rubber}{Area\ free\ to\ bulge}$$

$$S = \frac{\frac{\pi D^2}{4}}{\pi D t} = \frac{D}{4t}$$: Circular bearing

$$S = \frac{\frac{\pi}{4}(D_2^2 - D_1^2)}{\pi(D_2 + D_1)t} = \frac{D_2 - D_1}{4t}$$: Circular hollow bearing

$$S = \frac{\frac{\pi}{4}(D_2^2 - D_1^2)}{\pi D_2 t} = \frac{D_2^2 - D_1^2}{4 D_2 t}$$: Lead rubber bearing

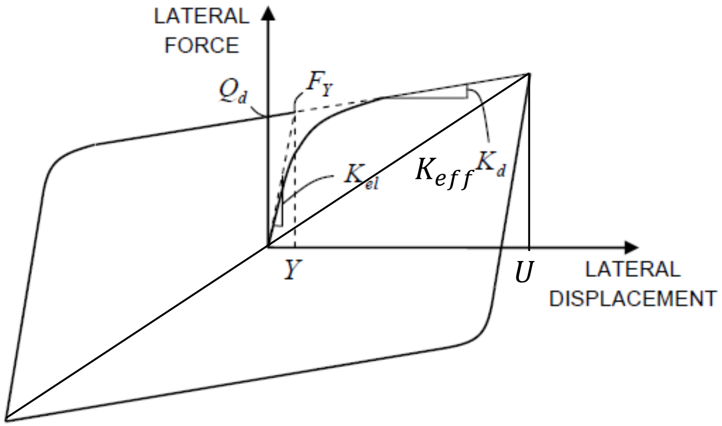


- Elastomer seismic isolation bearings have shape factors between 10 and 30.
- Small shape factor results in vertical flexible isolation bearings with small axial load capacity

Shear behavior

- Characterized by small shear stiffness

- Important parameters
  - Characteristics strength, Q_d
  - Yield strength, F_Y
  - Elastic stiffness, K_{el}
  - Post-elastic stiffness, K_d
  - Effective stiffness at displacement U, K_{eff}
  - Yield displacement, $Y \approx 0.05T_r - 0.1T_r$
  - Stiffness ratio, $\alpha = \frac{K_d}{K_{el}} \approx 0.1$
  - Effective damping ratios, β



Idealized shear behavior

Shear behavior

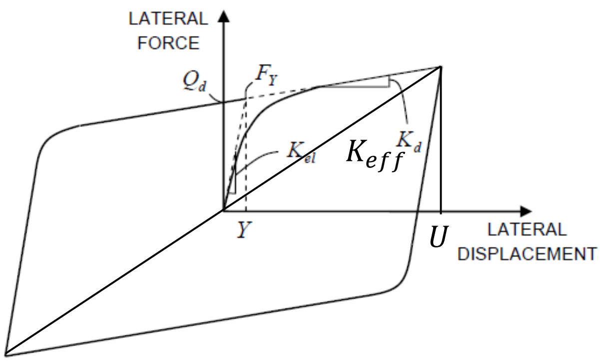
- Important relationships:

$$F_Y = \frac{Q_d}{1-\alpha}$$

$$\beta = \frac{Area\ under\ loop}{2\pi K_{eff}U^2} = \frac{4Q_d(U-Y)}{2\pi K_{eff}U^2}$$

$$Q_d = \frac{\pi\beta K_{eff}U^2}{2(U-Y)}$$

$$K_{eff} = \frac{F_{max}}{U} = \frac{Q_d + K_d U}{U} = \frac{Q_d}{U} + K_d$$



Idealized shear behavior

- For large values of displacement *U:* $K_{eff} \approx K_d$
- Preliminary sizing of bearing (discussed later) can be done using K_d without the need to obtain *U.*

Shear stiffness

- Experimental determination using force deformation loops under harmonic testing
- Force deformation loops can be
  - Viscoelastic
  - Hysteretic

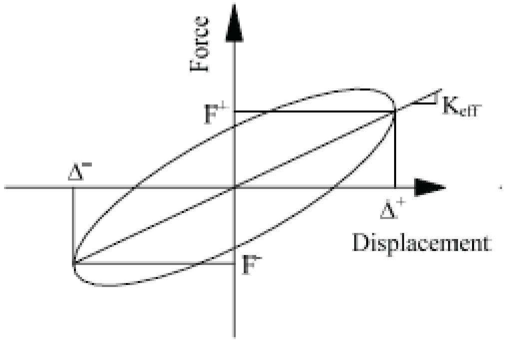
$$K_{eff} = \frac{|F^+| + |F^-|}{|\Delta^+| + |\Delta^-|}$$

- Shear modulus is determined using

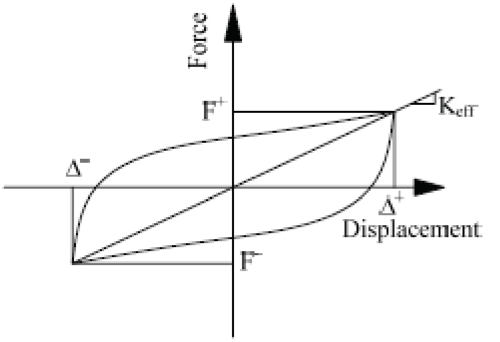
$$K_{eff} = \frac{G_{eff}A}{T_r}$$

A : bonded rubber area

- The effective shear modulus of natural rubber bearings for seismic isolation applications typically vary between 0.4-1.0 MPa



Viscoelastic behavior



Hysteretic behavior

Shear hysteresis

- The two horizontal directions are coupled
- Extension of Bouc-Wen model extended by Nagarajaiah et al. (1989) for seismic isolation applications:

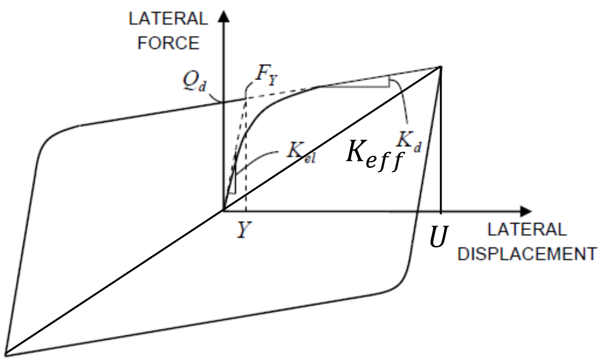
$$\begin{Bmatrix} F_x \\ F_y \end{Bmatrix} = c_d \begin{Bmatrix} \dot{U}_x \\ \dot{U}_y \end{Bmatrix} + K_d \begin{Bmatrix} U_x \\ U_y \end{Bmatrix} + (\sigma_{YL} A_L) \begin{Bmatrix} Z_x \\ Z_y \end{Bmatrix}$$

$$Y \begin{Bmatrix} \dot{Z}_x \\ \dot{Z}_y \end{Bmatrix} = \left(A[I] - \begin{bmatrix} Z_x^2 \left(\gamma Sign\left(\dot{U}_x Z_x\right) + \beta\right) & Z_x Z_y \left(\gamma Sign\left(\dot{U}_y Z_y\right) + \beta\right) \\ Z_x Z_y \left(\gamma Sign\left(\dot{U}_x Z_x\right) + \beta\right) & Z_y^2 \left(\gamma Sign\left(\dot{U}_y Z_y\right) + \beta\right) \end{bmatrix} \right) \begin{Bmatrix} \dot{U}_x \\ \dot{U}_y \end{Bmatrix}$$

- After yielding

$$A / (\beta + \gamma) = 1$$

$$Z_x = \cos\theta, \quad Z_y = \sin\theta$$

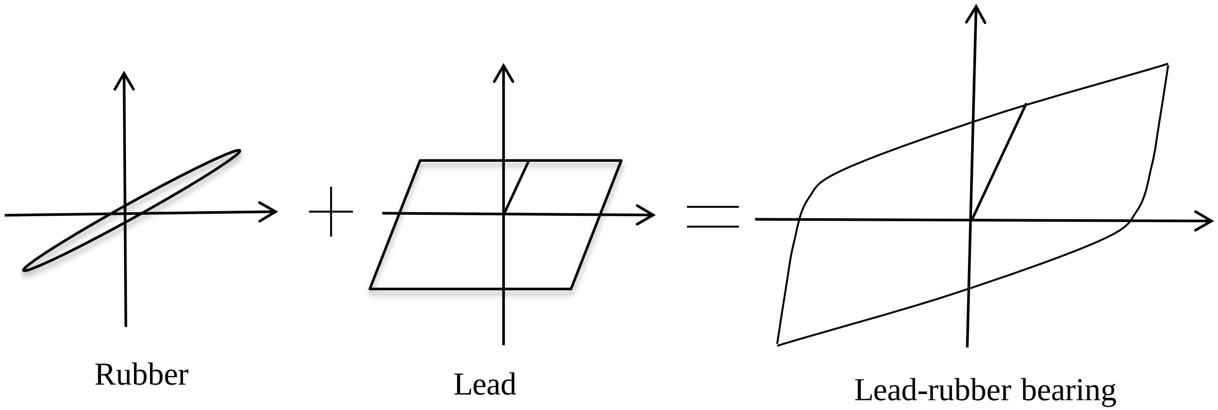


Idealized shear behavior

Shear hysteresis

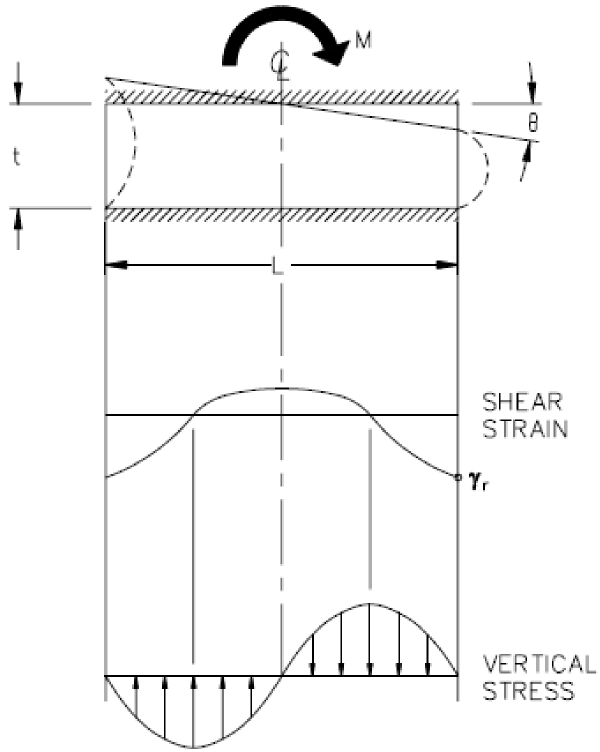
- Total horizontal force is the sum of rubber and hysteretic components

$$c_d \begin{Bmatrix}\dot{U}_x\\ \dot{U}_y\end{Bmatrix} + K_d \begin{Bmatrix}U_x\\ U_y\end{Bmatrix} + (\sigma_{YL} A_L)\begin{Bmatrix}Z_x\\ Z_y\end{Bmatrix} = \begin{Bmatrix}F_x\\ F_y\end{Bmatrix}$$



Torsional and rotational behavior

- For the rotation of circular and square bearings:
  - Rotational modulus: $E_r = \frac{E_c}{3}$
- Torsional and rotational behaviors of individual bearings are not going to affect the response of the isolation system.

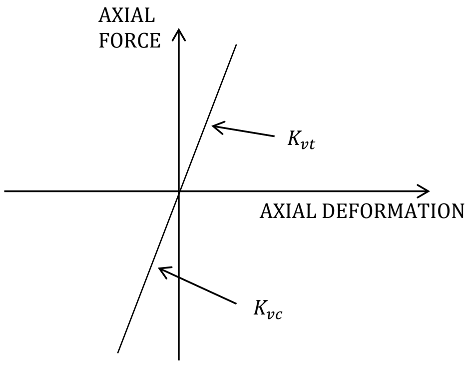


- Rotation
  - Linear elastic with stiffness = $\frac{IE_r}{T_r}$
- Torsion
  - Linear elastic with stiffness = $\frac{GJ}{T_r}$

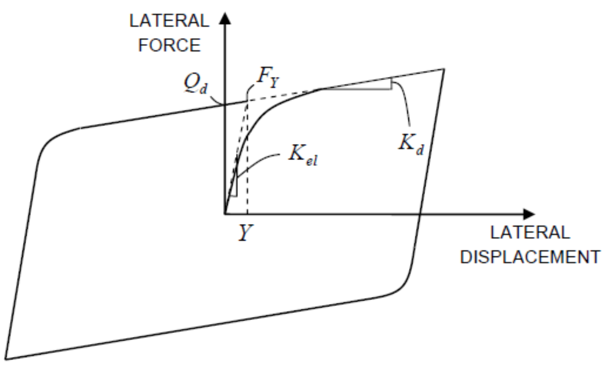
ADVANCED ISOLATOR MODELS

Advanced models

- Modeling challenges
  - Axial
    - Cavitation in tension
    - Buckling capacity in compression
  - Shear
    - Strength degradation in LR bearing
    - Coupling of horizontal motions
    - Axial load dependent stiffness

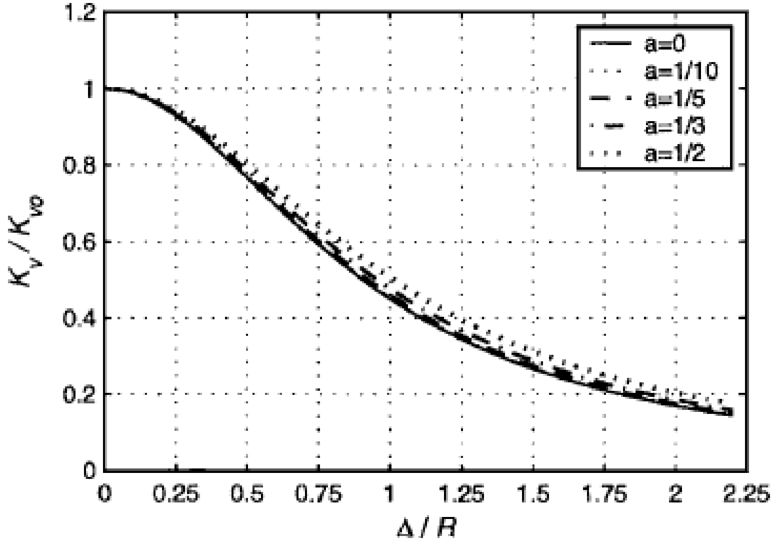


Idealized axial behavior

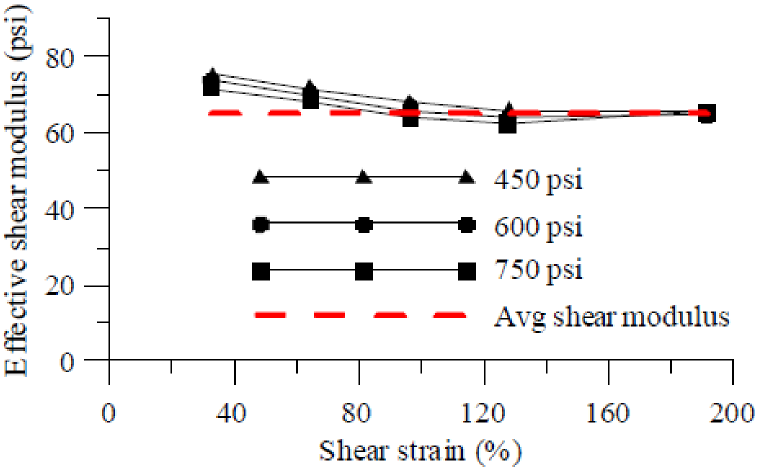


Idealized shear behavior

Advanced models

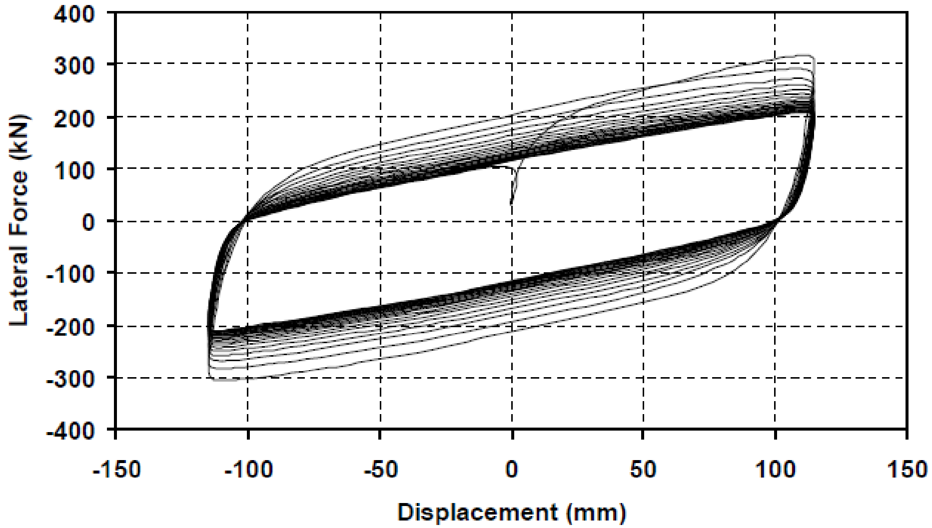


Varying vertical stiffness (Warn and Whittaker, 2006)

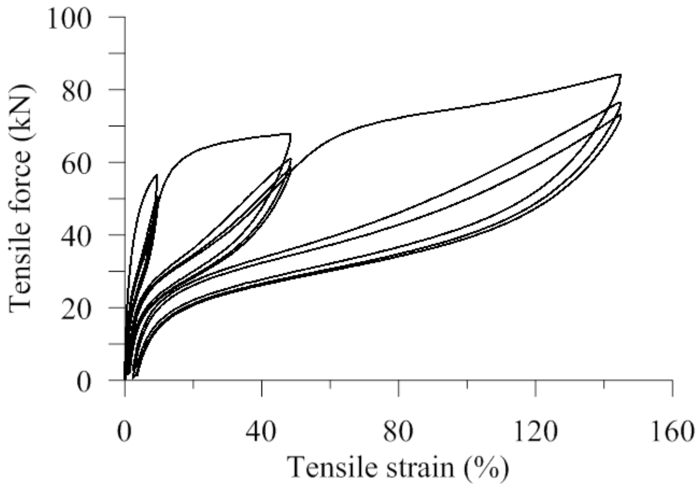


Stress and strain dependency of shear modulus (DIS, Inc.)

Advanced models



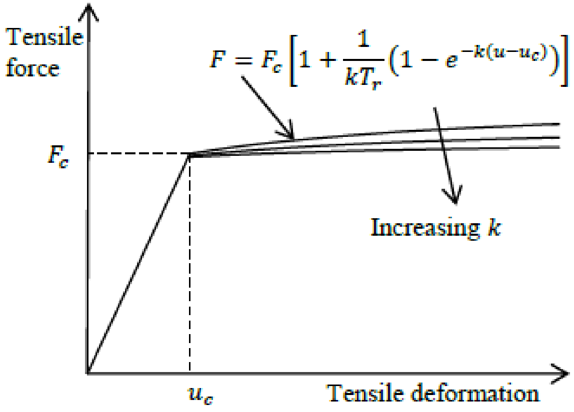
Strength degradation in LR bearing (Kalpakidis and Constantinou, 2008)



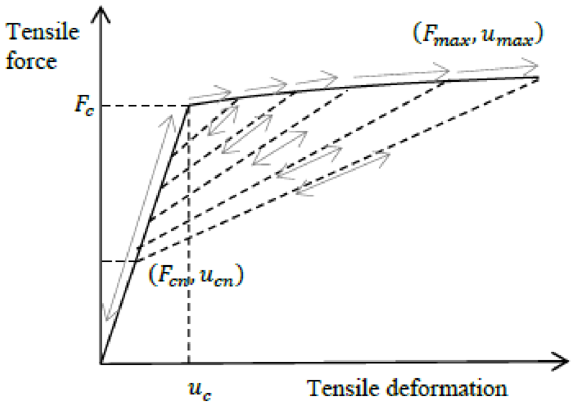
Cavitation in tension due to uplift and rocking (Warn, 2006)

Advanced models: tension

- A new phenomenological model
  - Pre-cavitation
    - Same as in compression
  - Post-cavitation behavior
    - Concept of “true area”
    - $\partial A/\partial u \propto A$
  - Permanent damage
    - Strain dependent damage index
    - $F_{cn} = F_c(1 - \emptyset)$
    - $\emptyset = \emptyset(u_{\max})$



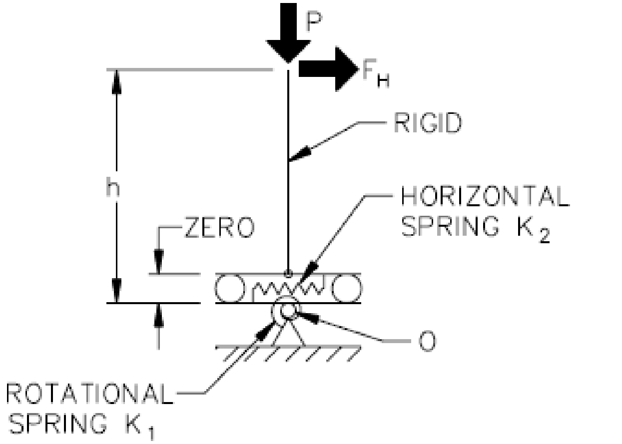
Post-cavitation behavior



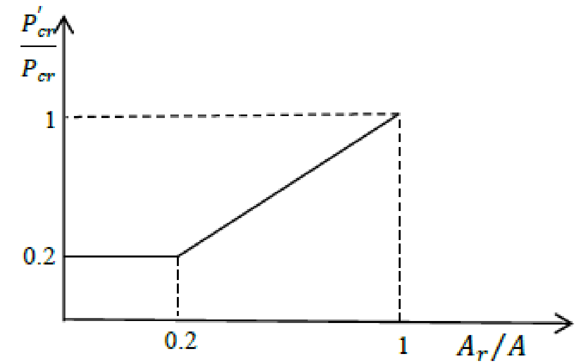
Strength-degradation in cyclic tension

Advanced models: compression

- Based on two-spring model
- Axial stiffness
  - Depends on shear deformation
- Critical buckling load
  - Bi-linear area reduction method
  - Validated by Warn et al.(2006)



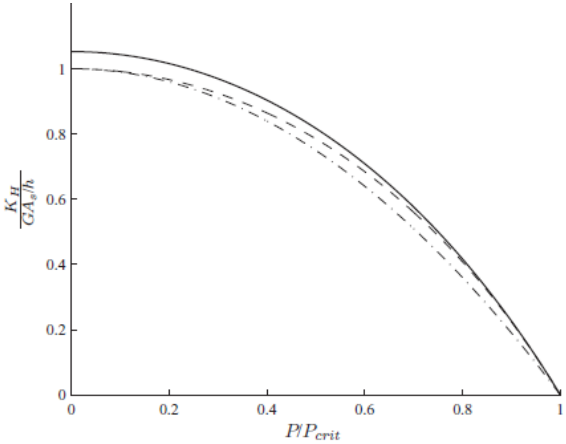
Two-spring model (Constantinou et al., 2007)



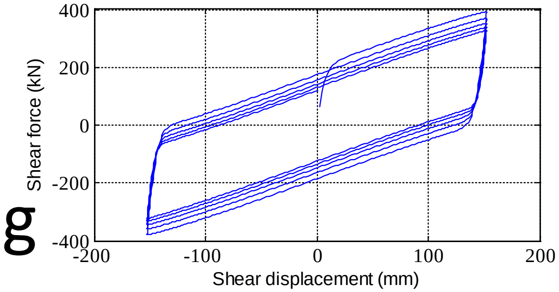
Bi-linear area reduction method

Advanced models: shear

- Bouc-Wen model for isolators
  - Nagarajaiah et al.(1991)
- Horizontal stiffness
  - $K_H = K_{H0}\left(1 - \left(\frac{P}{P_{cr}}\right)^2\right)$
- Strength degradation
  - Heating of lead-core in LR bearing
  - Based on Kalpakidis et al. (2010)

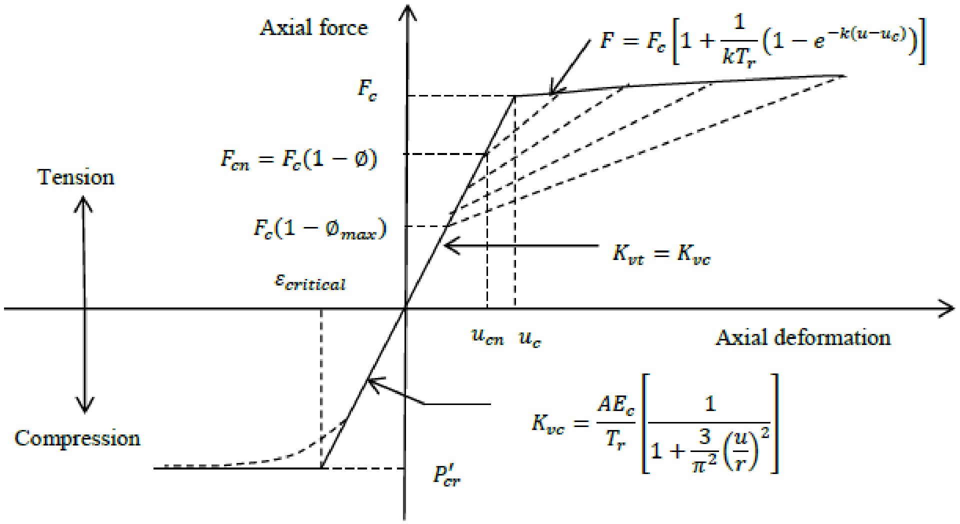


Horizontal stiffness (Kelly, 1993)

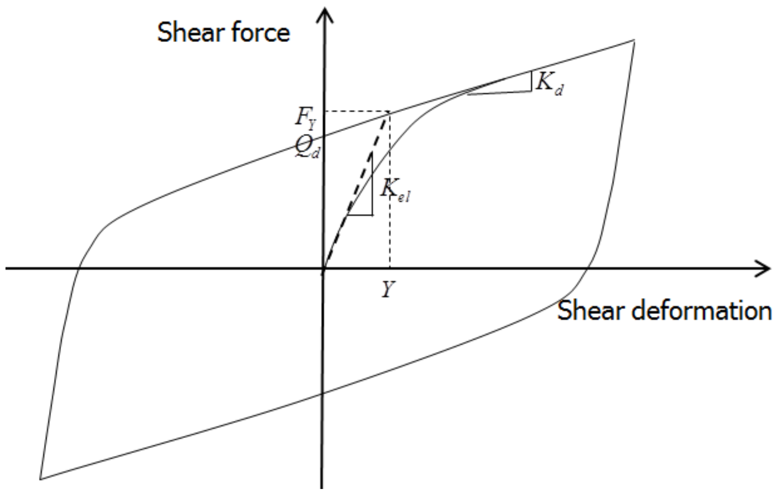


Strength degradation in a LR bearing

Advanced models: summary



Axial (vertical) direction



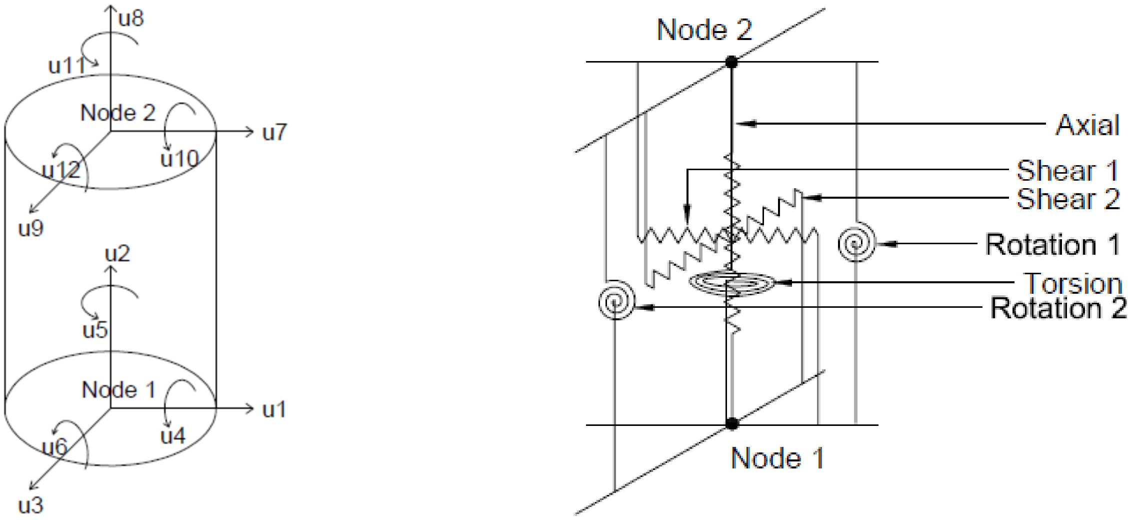
Shear (horizontal) direction

Implementation

- User elements in OpenSees and ABAQUS
  - Low damping rubber bearing: ElastomericX
  - Lead rubber bearing: LeadRubberX
  - High damping rubber bearing: HDRX
- Implementation in LS-DYNA as user material
  - Addition to *MAT_SEISMIC_ISOLATOR
- Input parameters
  - Geometric and material properties
  - Default values of optional parameters provided

Implementation

- Physical model
  - 2 Node, 12 DOF, 3D discrete element
  - Linear springs in rotational direction



Implementation

- User Elements (UELs)
  - Requires nodal force vector and stiffness matrix
  - Allows parameter update

$$F = \begin{bmatrix} Axial \\ Shear1 \\ Shear2 \\ Torsion \\ Rotation1 \\ Rotation2 \end{bmatrix} \quad K = \begin{bmatrix} Axial & 0 & 0 & 0 & 0 & 0 \\ 0 & Shear1 & Shear12 & 0 & 0 & 0 \\ 0 & Shear21 & Shear2 & 0 & 0 & 0 \\ 0 & 0 & 0 & Torsion & 0 & 0 \\ 0 & 0 & 0 & 0 & Rotation1 & 0 \\ 0 & 0 & 0 & 0 & 0 & Rotation2 \end{bmatrix}$$

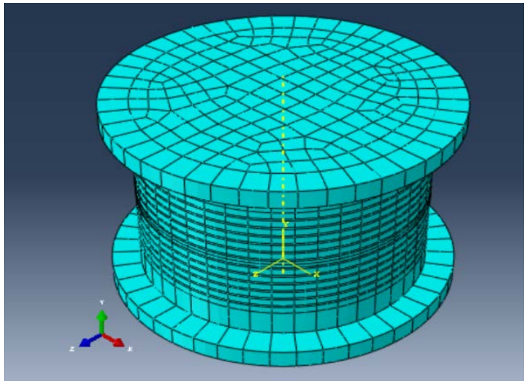
Advanced models: comparison

Properties	3DBASIS	SAP2000	PERFORM3D	LSDYNA	ABAQUS	OpenSees	New
Coupled horizontal directions	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Coupled horizontal and vertical directions	No	No	No	No	No	No	Yes
Different tensile and compressive stiffness	No	No	Yes	Yes	Yes	Yes	Yes
Nonlinear tensile behavior	No	No	No	No	Yes	Yes	Yes
Cavitation and post-cavitation	No	No	No	No	No	No	Yes
Nonlinear compressive behavior	No	No	No	No	Yes	Yes	Yes
Varying buckling capacity	No	No	No	No	No	No	Yes
Heating of lead core	No	No	No	No	No	No	Yes

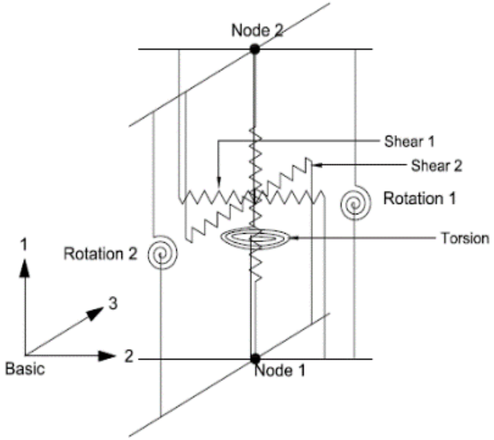
ANALYSIS USING CONTEMPORARY SOFTWARE PROGRAMS

Axial behavior: rubber isolator

- Analysis procedure in two software programs
  - SAP2000: discrete model
  - ABAQUS: FE model
- Modal analysis
- Response history analysis



Continuum FE model

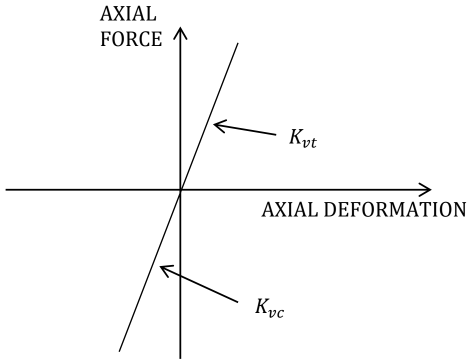


Discrete model

DISCRETE MODEL: SAP2000

Axial behavior: rubber isolator

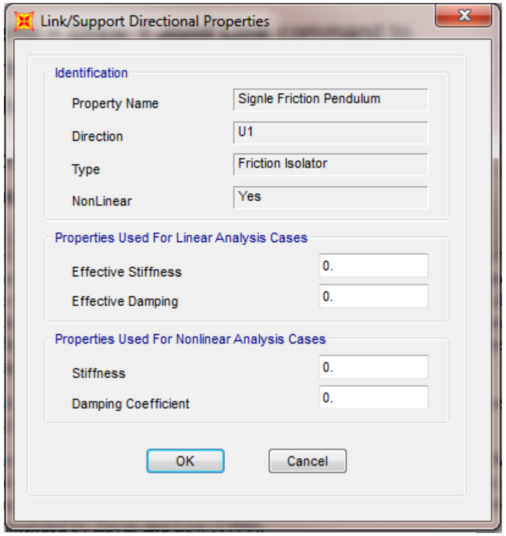
- U1 directional property
  - Always linear
- Effective stiffness: options
  - Make it fixed (check fixed box)
  - Assign a large value
    - Note: don't assign unrealistic large values
  - Calculate from bearing properties
    - $K_v = \frac{AE_c}{T_r}$ (See Constantinou et al (2007) for details)
- Effective damping
  - It is damping coefficient c_d and not the damping ratio ξ
  - Usually a value of 0 is recommended
  - If required c_d corresponding to 2-3% of damping ratio can be used for natural rubber

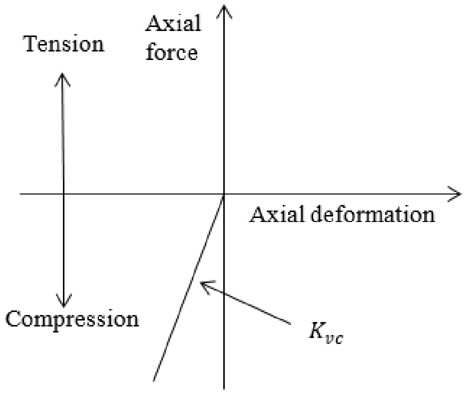


Axial behavior

Axial behavior: friction isolator

- U1 directional property
  - Always nonlinear
  - Compression only (gap)
- Effective stiffness: options
  - Make it fixed (check fixed box)
  - Assign a reasonably large value
    - 1000 times the horizontal stiffness

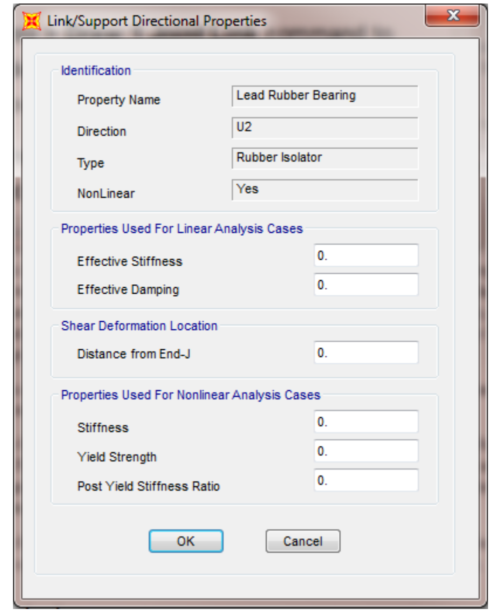




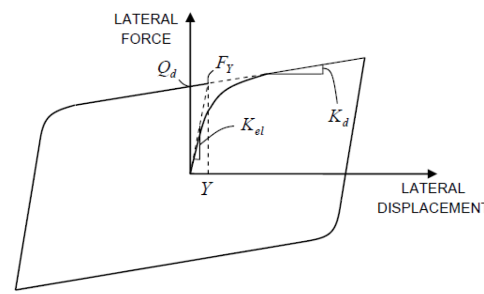
Axial behavior

Shear behavior: rubber isolator

- U2, U3 directional properties
  - Coupled bidirectional Bouc-Wen
- Mechanical properties
  - Calculate from bearing properties
    - See Constantinou et al (2007) for details



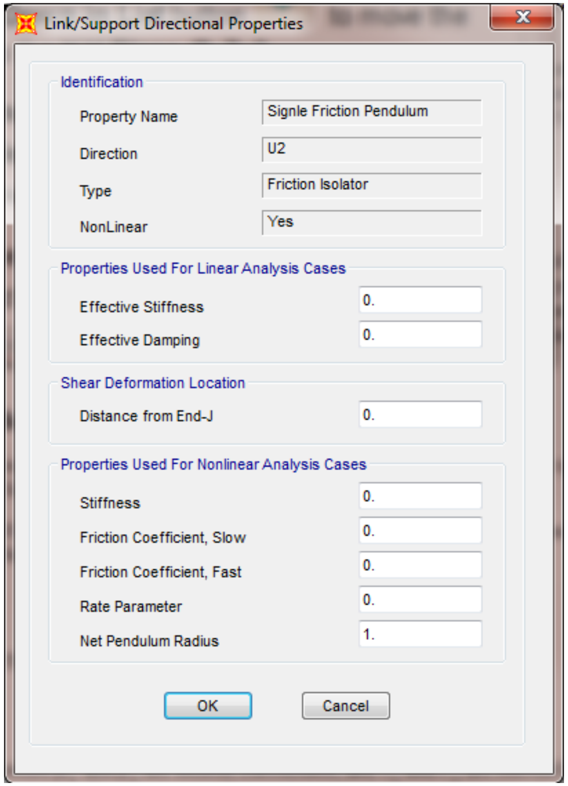
- Stiffness = K_{el}
- Yield strength = F_Y
- Post Yield Stiffness Ratio = K_d/K_{el}



Shear behavior

Shear behavior: friction isolator

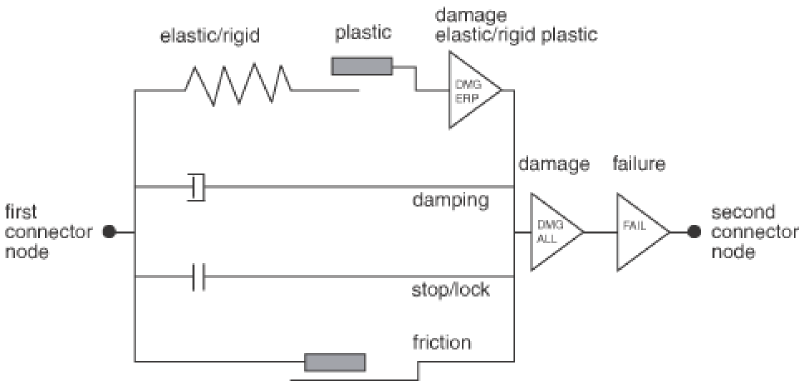
- U2, U3 directional properties
  - Coupled bidirectional Bouc-Wen
- Effective stiffness: options
  - Calculate from bearing properties
    - See Constantinou et al (2007) for details
- Friction can be varied
  - $\mu = \mu_{fast} - (\mu_{fast} -$



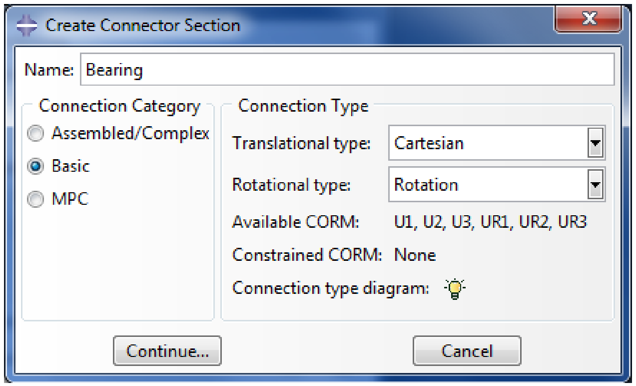
DISCRETE MODEL: ABAQUS

ABAQUS: Connector Element

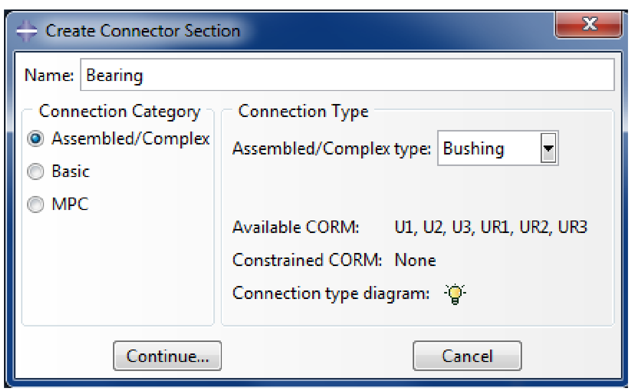
- Connector element
  - Similar to the Link/support element
- Elastic spring, dashpot, friction, plasticity, and damage
- Different directions between two nodes can be coupled, uncoupled or combined



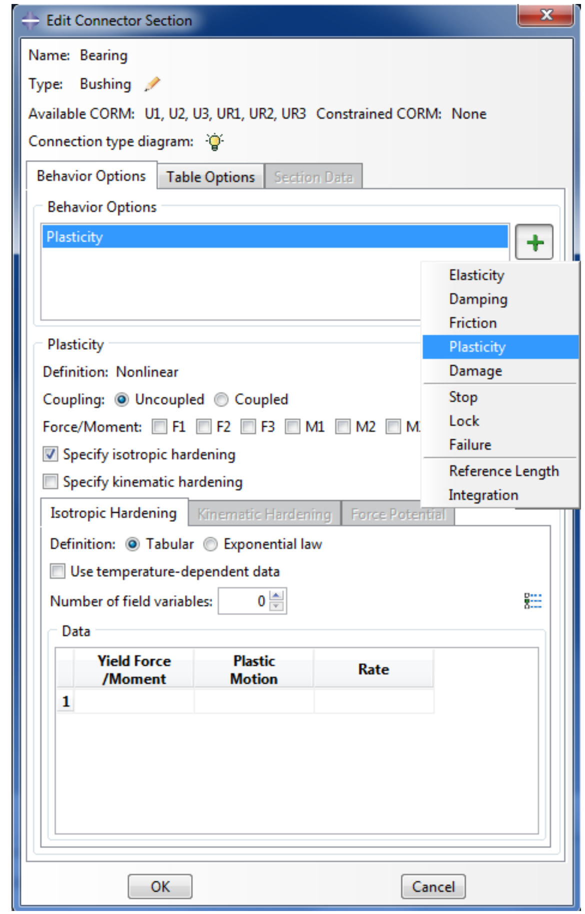
Connector definition



Basic connection



Assembled connection



Connector material behavior

DISCRETE MODEL: LS-DYNA

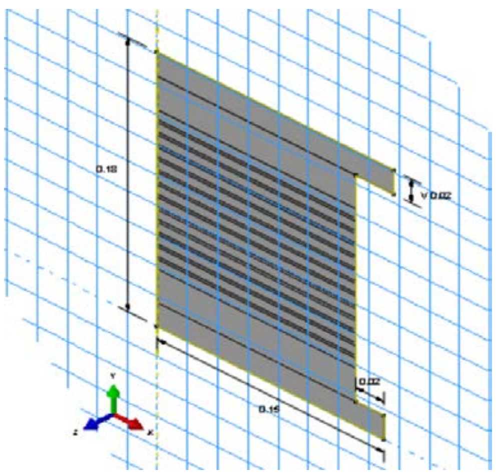
LSDYNA: MAT_SEISMIC_ISOLATOR

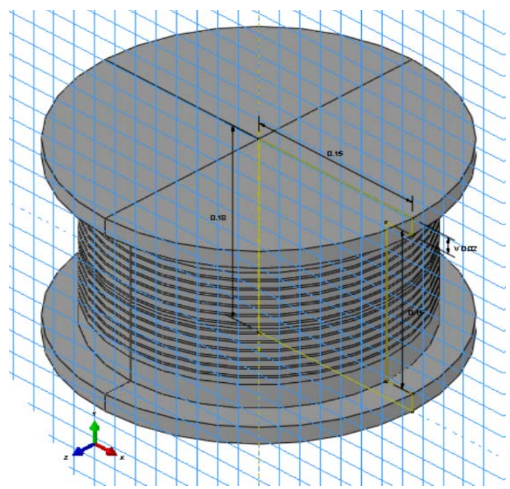
- *MAT_SEISMIC_ISOLATOR (*MAT_197)
- Can be used to model
  - Elastomeric bearings
  - Flat slider bearings, single FP bearings
  - Double FP bearings and XY-FP bearings

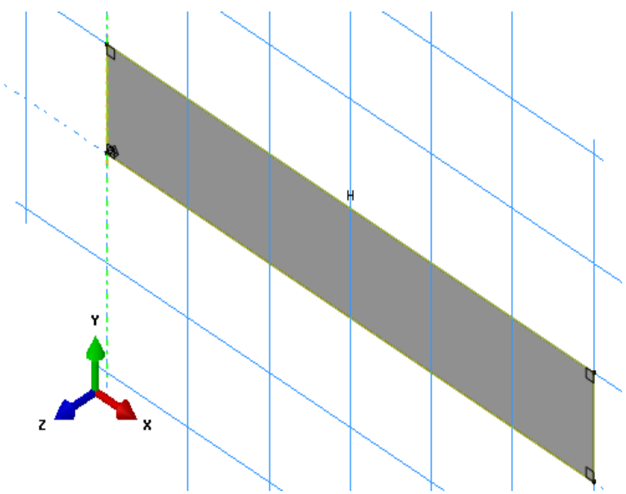
CONTINUUM MODEL

Finite element models

- Only component level analysis is presented
- Three FE modeling approaches
  - 3D model of elastomeric bearing
  - Axisymmetric model of elastomeric bearing
  - Axisymmetric model of single rubber layer

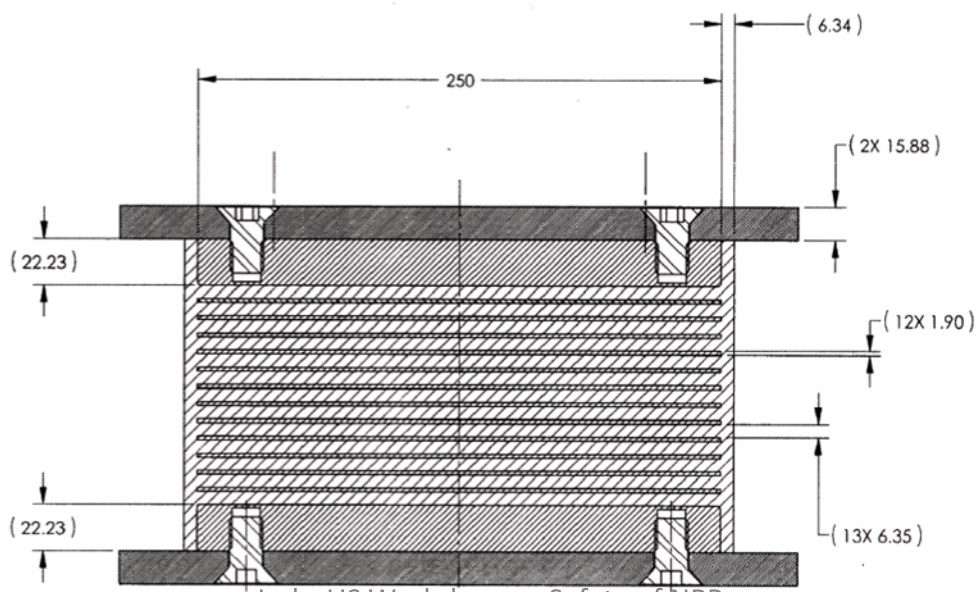






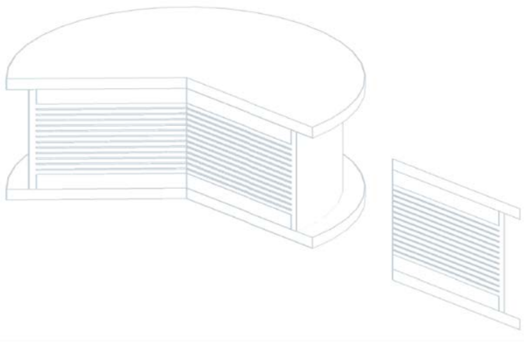
Geometry

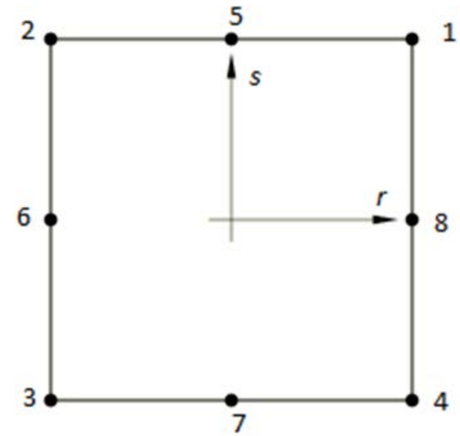
- Diameter D = 250 mm
- Total Rubber Thickness T_r = 82.5 mm
- Shape factor S = 9.8



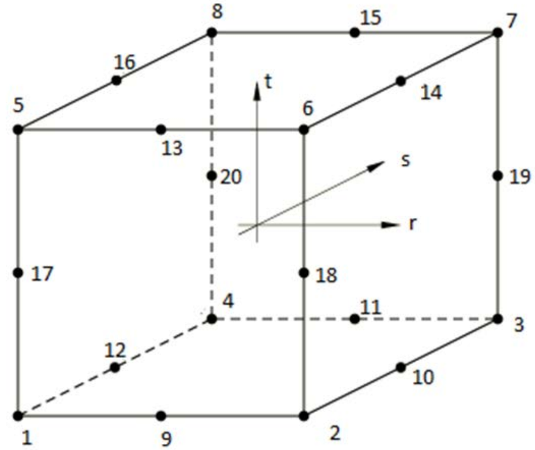
Elements

- Axisymmetric
  - CAX8R





- Three-dimensional
  - C3D20R



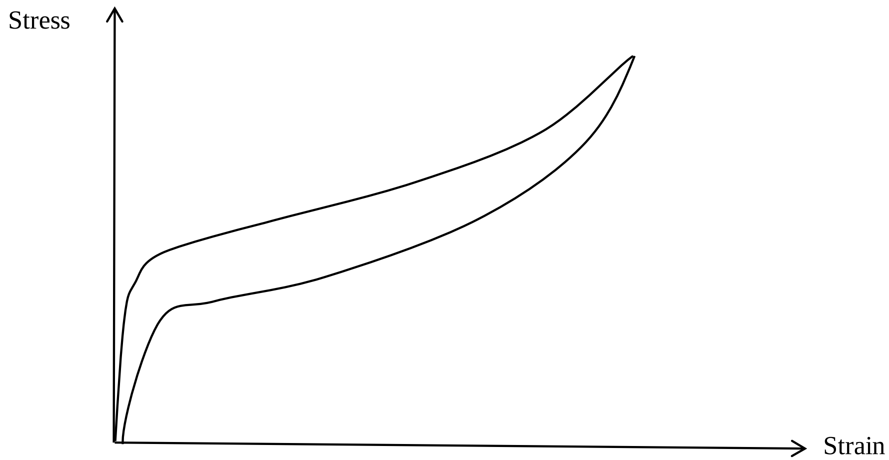
Materials

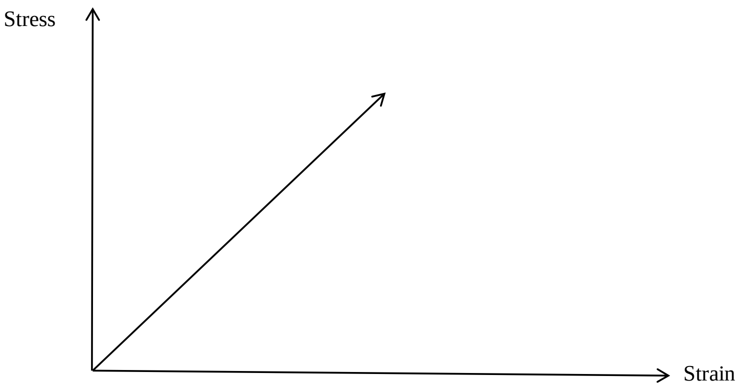
Rubber: hyperelastic

- Neo-Hookean (anisotropic)
- High bulk modulus (2000 Mpa)
- Low shear modulus (0.65 Mpa)

Steel: linear elastic

- Isotropic
- Young's Modulus: 210 Gpa
- Poisson's ratio: 0.3





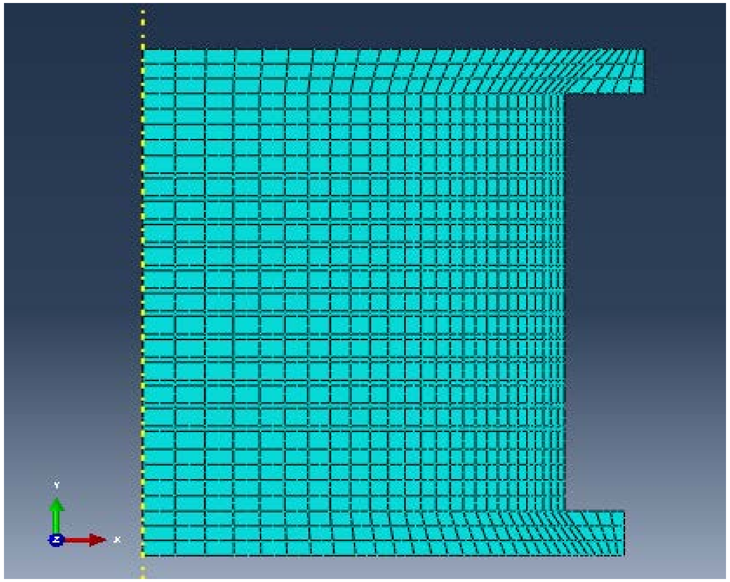
Meshing

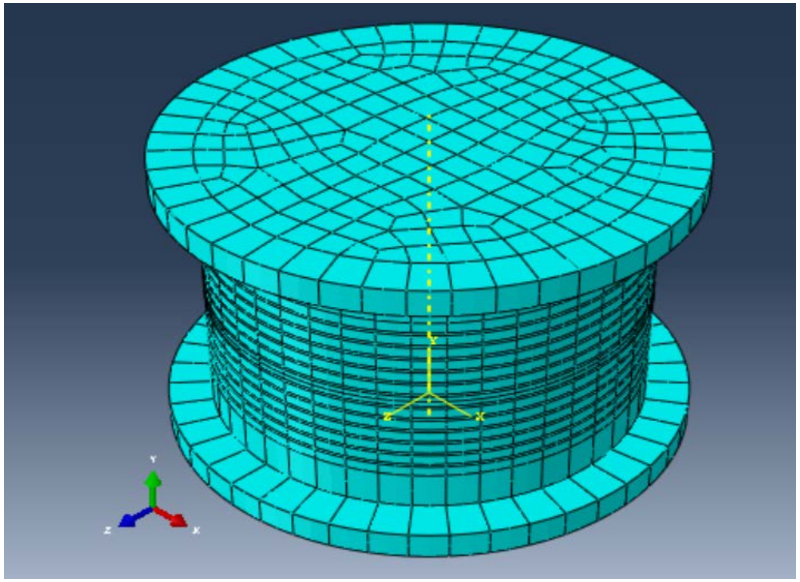
Axisymmetric model

- Mesh bias
- Structured meshing
- Quad elements

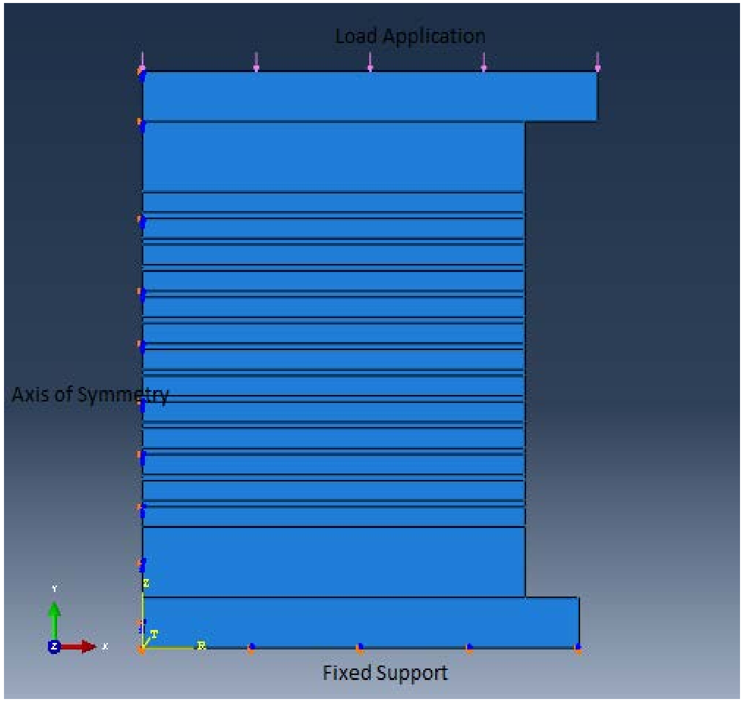
3D model

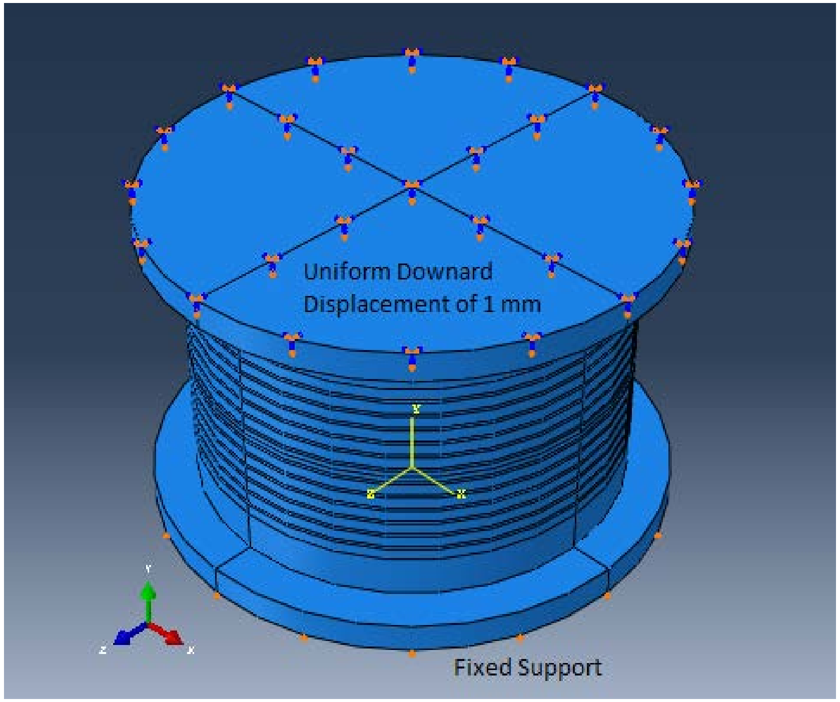
- Divided into sub-regions
- Swept meshing
- Hex only elements



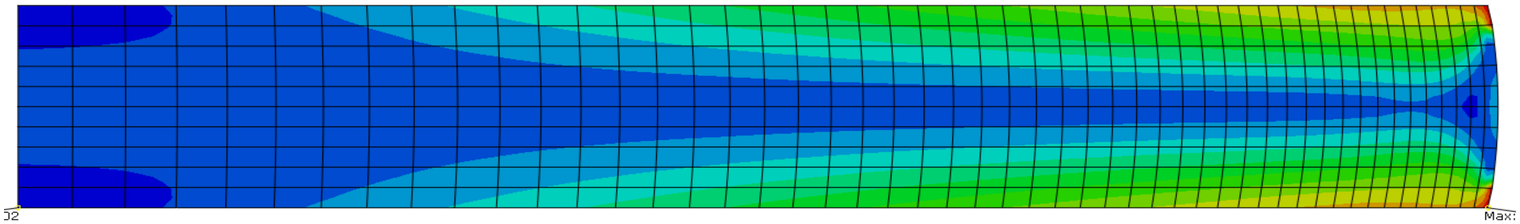


Boundary conditions

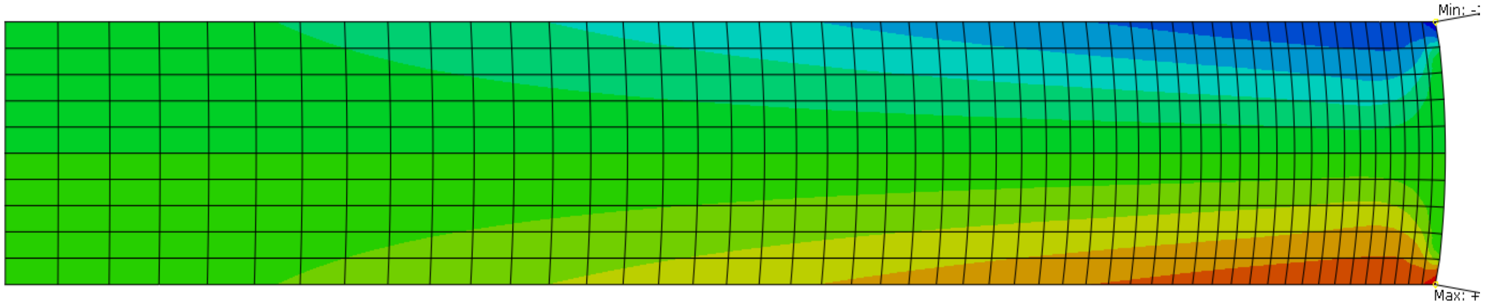




Results: Axisymmetric



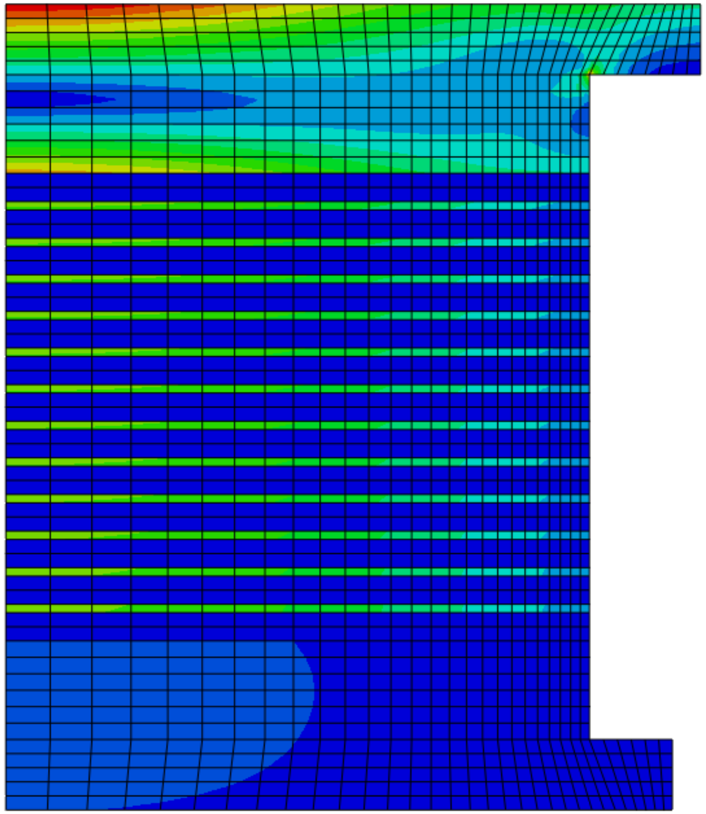
Stress state(Mises)



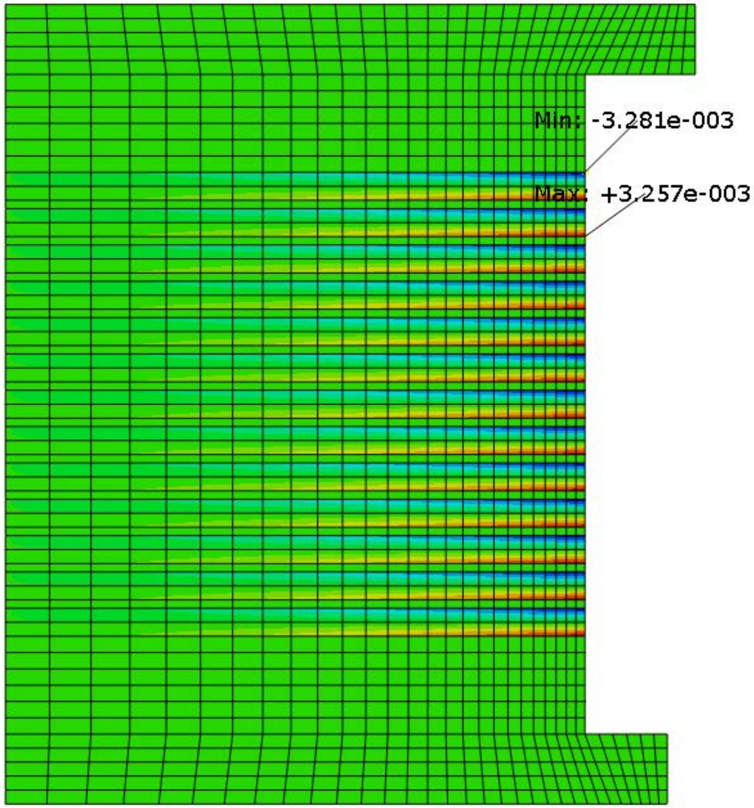
Logarithmic shear strain

Results: axisymmetric model

Stress(Mises)

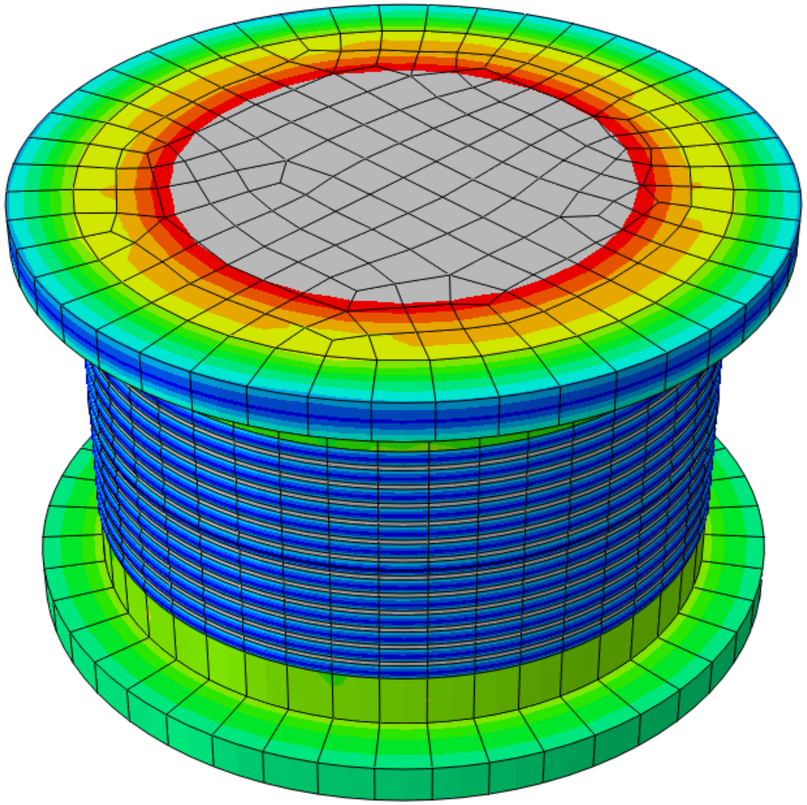


Logarithmic shear strain

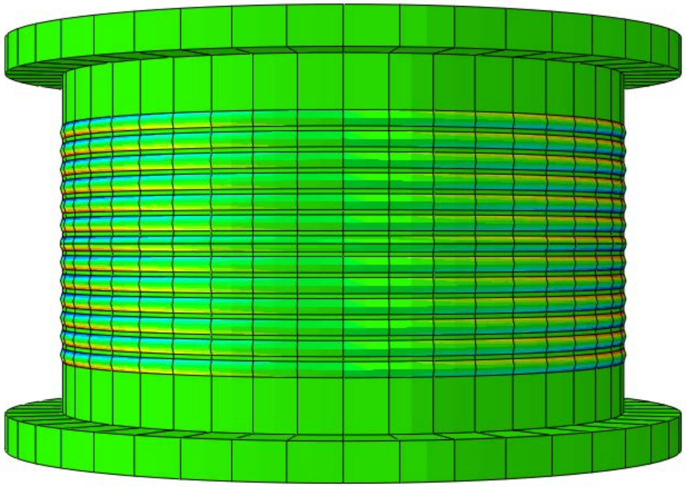


Results: 3D model

Stress(Von-Mises)



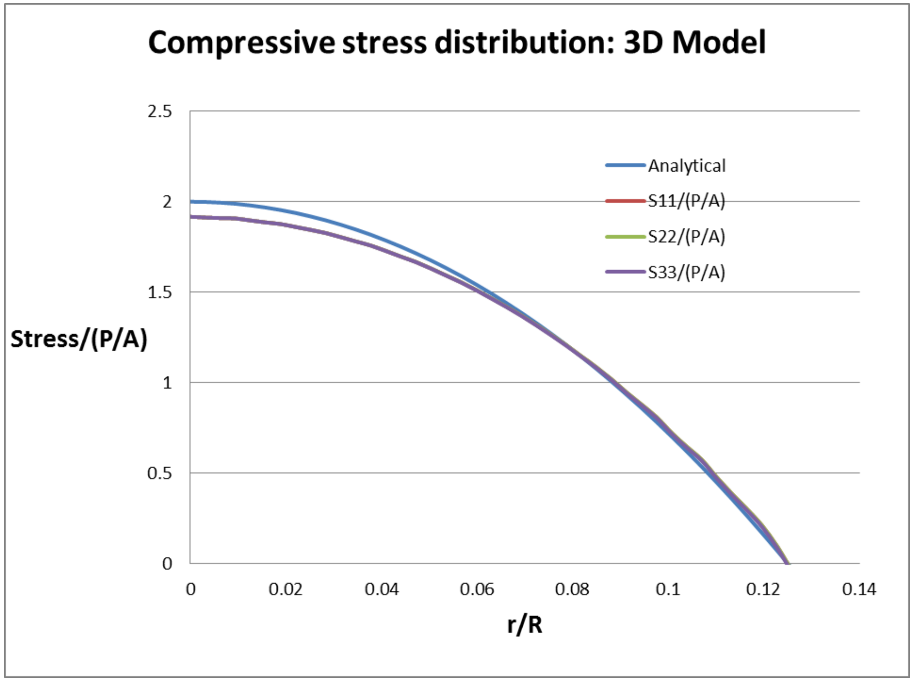
Logarithmic shear strain



Results comparison

Vertical stiffness in N/m

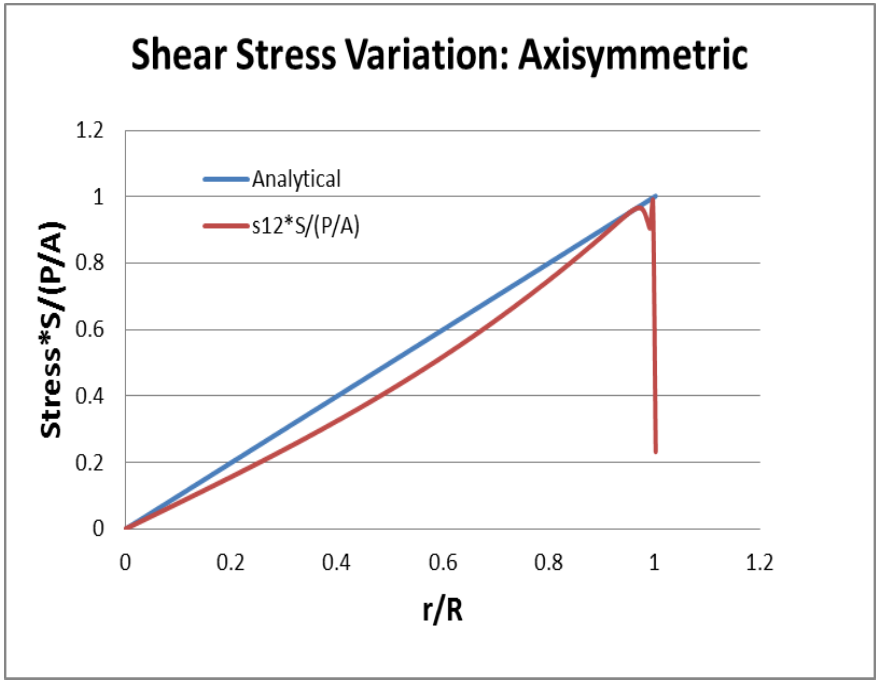
Analytical	Axisymmetric Model	3D Model
162.5×10^6	193.6×10^6	167.5×10^6

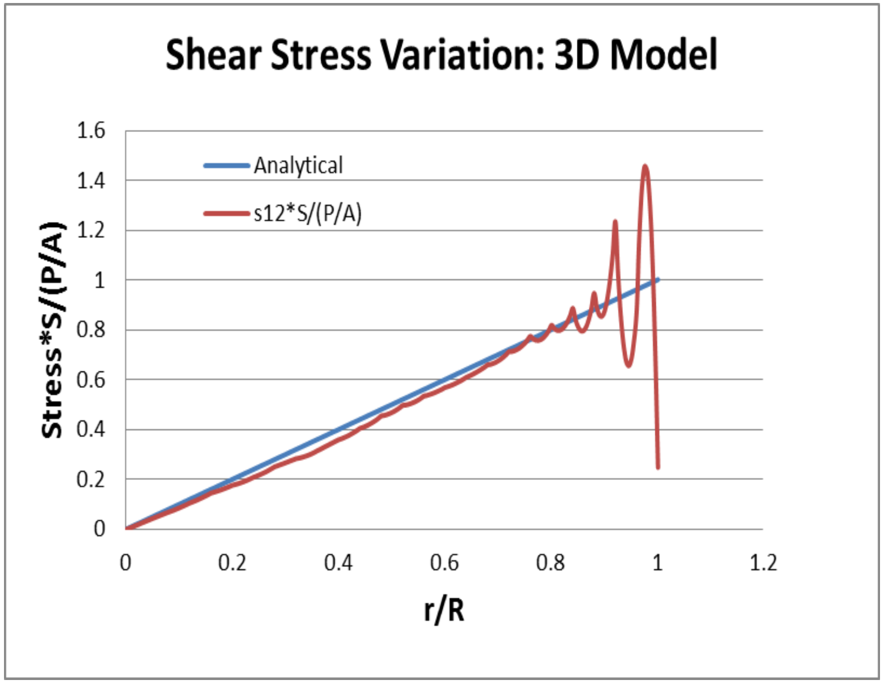


Normal stress distribution

Results comparison

- Shear stress along radius

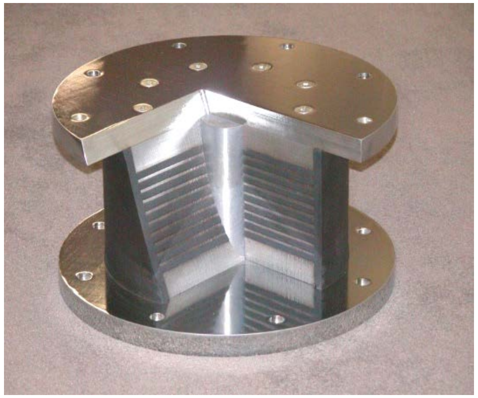




PRELIMINARY DESIGN

Key design parameters

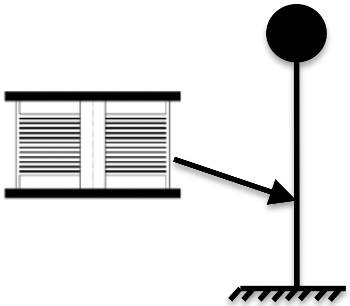
- Axial load capacity
  - Bearing should be able to sustain axial load with lateral response
- Lateral stiffness (shear modulus)
  - Determines the period of the isolation system
- Ultimate shear deformation capacity



Internal section view of lead-rubber bearing

Design procedure

- Many procedures are available for preliminary sizing of the bearings
- Two procedures would be discussed here
  - Non-iterative
  - Iterative
- Preliminary design can later be verified using dynamic analysis
- Preliminary design process remains same irrespective of structure type (e.g., buildings, bridges, others)
  - Structure is idealized as SDOF system with isolation period and lumped superstructure weight
  - Isolation mode must be the primary dominating mode (participation factor>90%)



Base-isolated building idealized as SDOF system

Preliminary sizing: non-iterative

- Known parameters
  - Seismic weight of the superstructure, *W*
  - Shear modulus of the bearing, *G (0.4-1.0 MPa)*
  - Nominal yield stress of lead for LR bearing, σ_L *(10-12 MPa)*
- Assumed parameters
  - Time period of the isolation system, *T* (between 2-4 s)
  - Strength to supported weight ratio, Q_d/W (between 0.06 to 0.15)
  - Allowable service static pressure on the bearing, p_{static} (4-8 MPa)
- Values of the parameters to be obtained
  - Outer diameter, D_o
  - Inner diameter (or lead core diameter), D_i
  - Total rubber thickness, T_r

Preliminary sizing: non-iterative

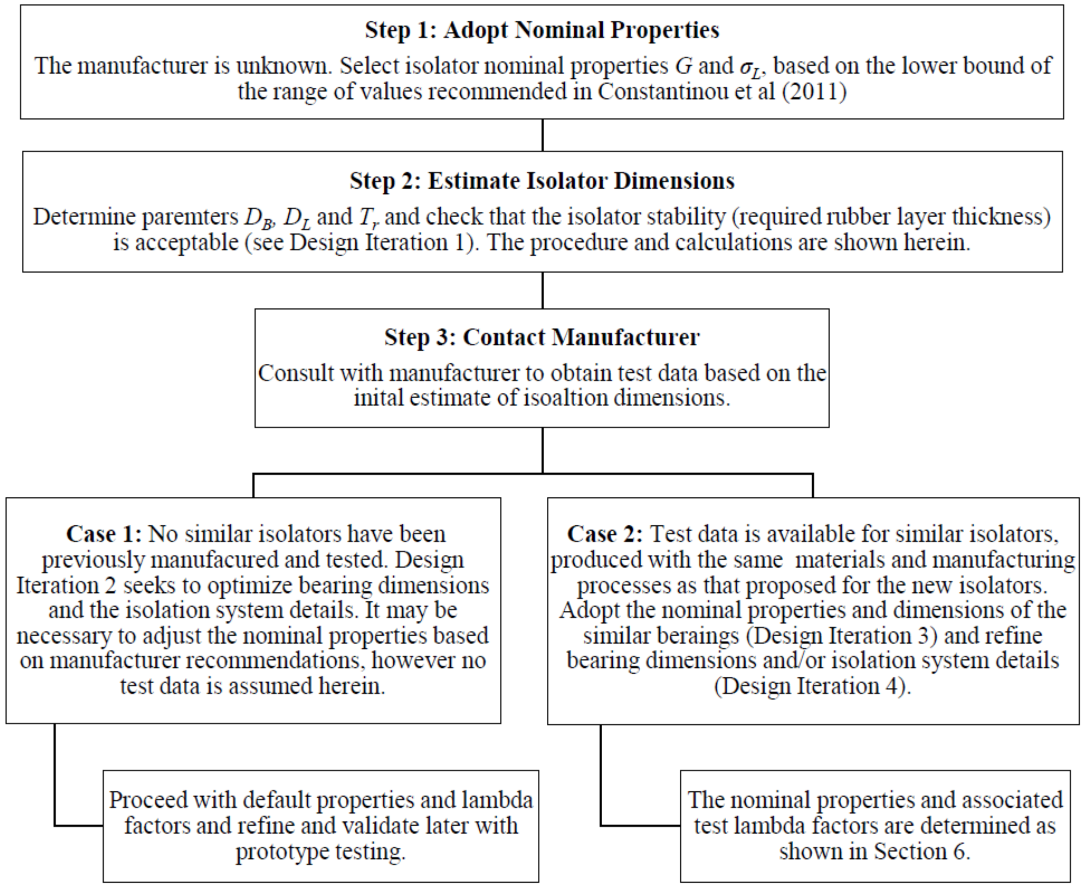
- The time period (*T*) and strength ratio (Q_d/W) completely characterizes the horizontal response of base-isolated structures, *TxQy*: *T2Q12.*
- Bearing manufactures would specify which parameter values can reliably be achieved and based on that other design parameters need to be obtained.
- Values of obtain design parameters need to be confirmed with manufacturer to ensure that bearings with these properties can be produced.

$$A_L = \frac{(Q_d / W) \times W}{\sigma_L}; D_i = \sqrt{4 \frac{A_L}{\pi}}$$

$$A = \frac{W}{p_{static}}; \quad D_o = \sqrt{\frac{4A}{\pi} + D_i^2} - t_c$$

$$M = \frac{W}{g}; K_{H0} = \frac{4\pi^2 M}{T^2}; T_r = \frac{GA}{K_{H0}}$$

Preliminary sizing: Iterative



Source: McVitty and Constantinou (2015)

Preliminary sizing: Iterative

- $D_i = \sqrt{4 \frac{(Q_d/W) \times W}{\pi \sigma_L}}$
- Assume a value of shear modulus, *G* (0.4-1.0 MPa)
- Diameter of bearing, D_o, and total rubber thickness, T_r, is selected based on lead core diameter D_i
  - D_o should be between $3D_i$ and $6D_i$
  - T_r should be equal to, or greater than D_i
- Lateral stiffness of the isolation system is obtained
  - $K_d = G \frac{\pi\left(D_o^2 - D_i^2\right)}{4T_r}$

Preliminary sizing: Iterative

- The MCE displacement of the system is obtained using an iterative equivalent lateral force procedure
  - Assume a MCE displacement, D_M
  - Calculate effective stiffness: $K_M = K_d + \frac{Q_d}{D_M}$
  - Effective isolation period: $T_M = 2\pi\sqrt{\frac{W}{gK_M}}$
  - Calculate effective damping: $\beta_M = \frac{4Q_d(D_M - Y)}{2\pi K_M D_M^2}$ (assume *Y* between 0.05 T_r to 0.1 T_r)
  - Obtain the damping reduction factor, B_M, from code (e.g., ASCE 7, Table 17.5-1)
  - Calculate the displacement using response spectrum: $D_M = \frac{gS_{M1}T_M}{4\pi^2 B_M}$
  - If the obtained displacement does not match the assumed MCE displacement, iterate with the new assumption as the average of the two values.

Summary and conclusions

- Several numerical tools are available for analysis of elastomeric bearings
- Finite element and discrete modeling approaches are used for the analysis of isolation bearings
- Discrete method is more popular owing to its simplicity and computational efficiency
- Preliminary design of bearings can be done using simplified procedures
- The current state of practice does not include advanced behavior that might be important under extreme earthquake shaking
- User element and materials of elastomeric seismic isolation bearings implemented in OpenSees, ABAQUS and LS-DYNA can capture complex behavior

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Towards a safer and resilient infrastructure

Thank You!

Questions?

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